

8

Cumulative Sum and Exponentially Weighted Moving Average Control Charts

CHAPTER OUTLINE

8-1 THE CUMULATIVE SUM CONTROL CHART

- 8-1.1 Basic Principles: The Cusum Control Chart for Monitoring the Process Mean
- 8-1.2 The Tabular or Algorithmic Cusum for Monitoring the Process Mean
- 8-1.3 Recommendations for Cusum Design
- 8-1.4 The Standardized Cusum
- 8-1.5 Improving Cusum Responsiveness for Large Shifts
- 8-1.6 The Fast Initial Response or Headstart Feature
- 8-1.7 One-Sided Cusums
- 8-1.8 A Cusum for Monitoring Process Variability
- 8-1.9 Rational Subgroups
- 8-1.10 Cusums for Other Sample Statistics
- 8-1.11 The V-Mask Procedure

8-2 THE EXPONENTIALLY WEIGHTED MOVING AVERAGE CONTROL CHART

- 8-2.1 The Exponentially Weighted Moving Average Control Chart for Monitoring the Process Mean
- 8-2.2 Design of an EWMA Control Chart
- 8-2.3 Robustness of the EWMA to Nonnormality
- 8-2.4 Rational Subgroups
- 8-2.5 Extensions of the EWMA

8-3 THE MOVING AVERAGE CONTROL CHART

Supplemental Material for Chapter 8

- S8-1 The Markov Chain Approach for Finding ARL for Cusum and EWMA Control Charts
- S8-2 Integral Equation versus Markov Chains for Finding the ARL

The supplemental material is on the textbook website www.wiley.com/college/montgomery.

LEARNING OBJECTIVES

1. Set up and use cusum control charts for monitoring the process mean
2. Design a cusum control chart for the mean to obtain specific ARL performance
3. Incorporate a fast initial response feature into the cusum control chart
4. Use a combined Shewhart-cusum monitoring scheme
5. Set up and use EWMA control charts for monitoring the process mean
6. Design an EWMA control chart for the mean to obtain specific ARL performance
7. Understand why the EWMA control chart is robust to the assumption of normality
8. Understand the performance advantage of cusum and EWMA control charts relative to Shewhart control charts
9. Set up and use a control chart based on an ordinary (unweighted) moving average

8-1 THE CUMULATIVE SUM CONTROL CHART

Table 8-1 Data for the Cusum Example

Sample, i	(a) x_i	(b) $x_i - 10$	(c) $C_i = (x_i - 10) + C_{i-1}$
1	9.45	-0.55	-0.55
2	7.99	-2.01	-2.56
3	9.29	-0.71	-3.27
4	11.66	1.66	-1.61
5	12.16	2.16	0.55
6	10.18	0.18	0.73
7	8.04	-1.96	-1.23
8	11.46	1.46	0.23
9	9.20	-0.80	-0.57
10	10.34	0.34	-0.23
11	9.03	-0.97	-1.20
12	11.47	1.47	0.27
13	10.51	0.51	0.78
14	9.40	-0.60	0.18
15	10.08	0.08	0.26
16	9.37	-0.63	-0.37
17	10.62	0.62	0.25
18	10.31	0.31	0.56
19	8.52	-1.48	-0.92
20	10.84	0.84	-0.08
21	10.90	0.90	0.82
22	9.33	-0.67	0.15
23	12.29	2.29	2.44
24	11.50	1.50	3.94
25	10.60	0.60	4.54
26	11.08	1.08	5.62
27	10.38	0.38	6.00
28	11.62	1.62	7.62
29	11.31	1.31	8.93
30	10.52	0.52	9.45

UCL = 13
Center line = 10
LCL = 7

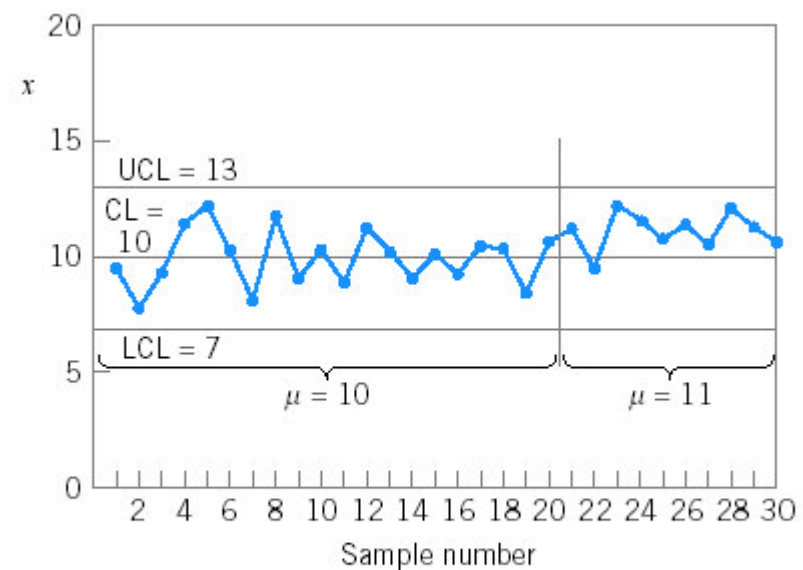


Figure 8-1 A Shewhart control chart for the data in Table 8-1.

The cusum chart directly incorporates all the information in the sequence of sample values by plotting the cumulative sums of the deviations of the sample values from a target value. For example, suppose that samples of size $n \geq 1$ are collected, and $\bar{x}_j$ is the average of the j th sample. Then if μ_0 is the target for the process mean, the cumulative sum control chart is formed by plotting the quantity

$$C_i = \sum_{j=1}^i (\bar{x}_j - \mu_0) \quad (8-1)$$

against the sample number i . C_i is called the cumulative sum up to and including the i th sample. Because they combine information from *several* samples, cumulative sum charts are more effective than Shewhart charts for detecting small process shifts.

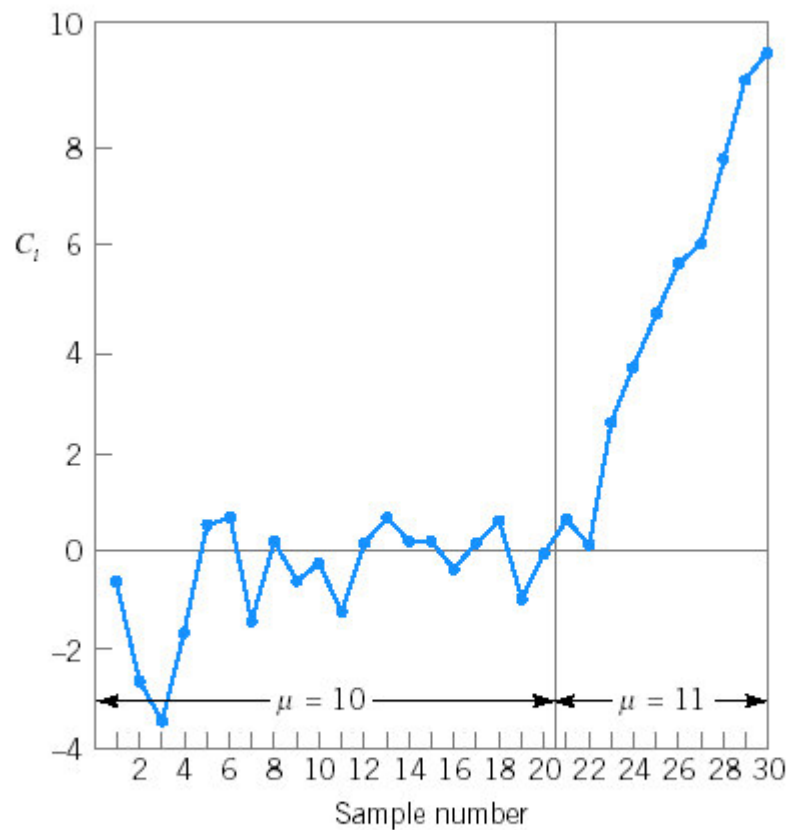


Figure 8-2 Plot of the cumulative sum from column (c) of Table 8-1.

$$\begin{aligned}
 C_i &= \sum_{j=1}^i (x_j - 10) \\
 &= (x_i - 10) + \sum_{j=1}^{i-1} (x_j - 10) \\
 &= (x_i - 10) + C_{i-1}
 \end{aligned}$$

The Tabular Cusum

$$C_i^+ = \max\left[0, x_i - (\mu_0 + K) + C_{i-1}^+\right] \quad (8-2)$$

$$C_i^- = \max\left[0, (\mu_0 - K) - x_i + C_{i-1}^-\right] \quad (8-3)$$

where the starting values are $C_0^+ = C_0^- = 0$.

$$K = \frac{\delta}{2}\sigma = \frac{|\mu_1 - \mu_0|}{2} \quad (8-4)$$

..... **EXAMPLE 8-1**

A Tabular Cusum

We will demonstrate the calculations for the tabular cusum by using the data from Table 8-1. Recall that the target value is $\mu_0 = 10$, the subgroup size is $n = 1$, the process standard deviation is $\sigma = 1$, and suppose that the magnitude of the shift we are interested in detecting is $1.0\sigma = 1.0(1.0) = 1.0$. Therefore, the out-of-control value of the process mean is $\mu_1 = 10 + 1 = 11$. We will use a tabular cusum with $K = \frac{1}{2}$ (because the shift size is 1.0σ and $\sigma = 1$) and $H = 5$ (because the recommended value of the decision interval is $H = 5\sigma = 5(1) = 5$).

Table 8-2 presents the tabular cusum scheme. To illustrate the calculations, consider period 1. The equations for C_i^+ and C_i^- are

$$C_1^+ = \max[0, x_1 - 10.5 + C_0^+]$$

and

$$C_1^- = \max[0, 9.5 - x_1 + C_0^-]$$

since $K = 0.5$ and $\mu_0 = 10$. Now $x_1 = 9.45$, so since $C_0^+ = C_0^- = 0$,

$$C_1^+ = \max[0, 9.45 - 10.5 + 0] = 0$$

and

$$C_1^- = \max[0, 9.5 - 9.45 + 0] = 0.05$$

Table 8-2 The Tabular Cusum for Example 8-1

Period i	x_i	(a)			(b)		
		$x_i - 10.5$	C_i^+	N^+	$9.5 - x_i$	C_i^-	N^-
1	9.45	-1.05	0	0	0.05	0.05	1
2	7.99	-2.51	0	0	1.51	1.56	2
3	9.29	-1.21	0	0	0.21	1.77	3
4	11.66	1.16	1.16	1	-2.16	0	0
5	12.16	1.66	2.82	2	-2.66	0	0
6	10.18	-0.32	2.50	3	-0.68	0	0
7	8.04	-2.46	0.04	4	1.46	1.46	1
8	11.46	0.96	1.00	5	-1.96	0	0
9	9.20	-1.3	0	0	0.30	0.30	1
10	10.34	-0.16	0	0	-0.84	0	0
11	9.03	-1.47	0	0	0.47	0.47	1
12	11.47	0.97	0.97	1	-1.97	0	0
13	10.51	0.01	0.98	2	-1.01	0	0
14	9.40	-1.10	0	0	0.10	0.10	1
15	10.08	-0.42	0	0	-0.58	0	0
16	9.37	-1.13	0	0	0.13	0.13	1
17	10.62	0.12	0.12	1	-1.12	0	0
18	10.31	-0.19	0	0	-0.81	0	0
19	8.52	-1.98	0	0	0.98	0.98	1
20	10.84	0.34	0.34	1	-1.34	0	0
21	10.90	0.40	0.74	2	-1.40	0	0
22	9.33	-1.17	0	0	0.17	0.17	1
23	12.29	1.79	1.79	1	-2.79	0	0
24	11.50	1.00	2.79	2	-2.00	0	0
25	10.60	0.10	2.89	3	-1.10	0	0
26	11.08	0.58	3.47	4	-1.58	0	0
27	10.38	-0.12	3.35	5	-0.88	0	0
28	11.62	1.12	4.47	6	-2.12	0	0
29	11.31	0.81	5.28	7	-1.81	0	0
30	10.52	0.02	5.30	8	-1.02	0	0

For period 2, we would use

$$\begin{aligned}C_2^+ &= \max[0, x_2 - 10.5 + C_1^+] \\ &= \max[0, x_2 - 10.5 + 0]\end{aligned}$$

and

$$\begin{aligned}C_2^- &= \max[0, 9.5 - x_2 + C_1^-] \\ &= \max[0, 9.5 - x_2 + 0.05]\end{aligned}$$

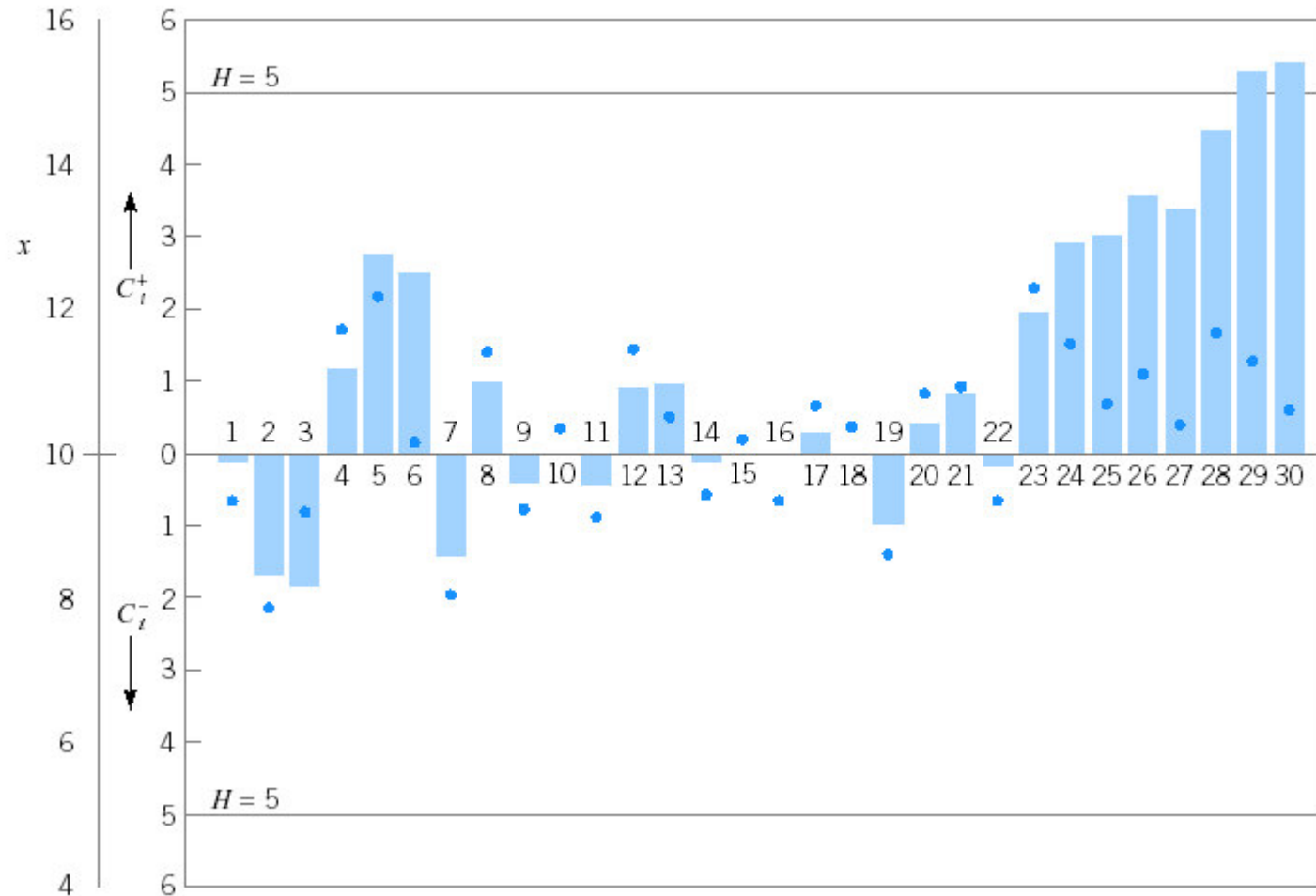
Since $x_2 = 7.99$, we obtain

$$C_2^+ = \max[0, 7.99 - 10.5 + 0] = 0$$

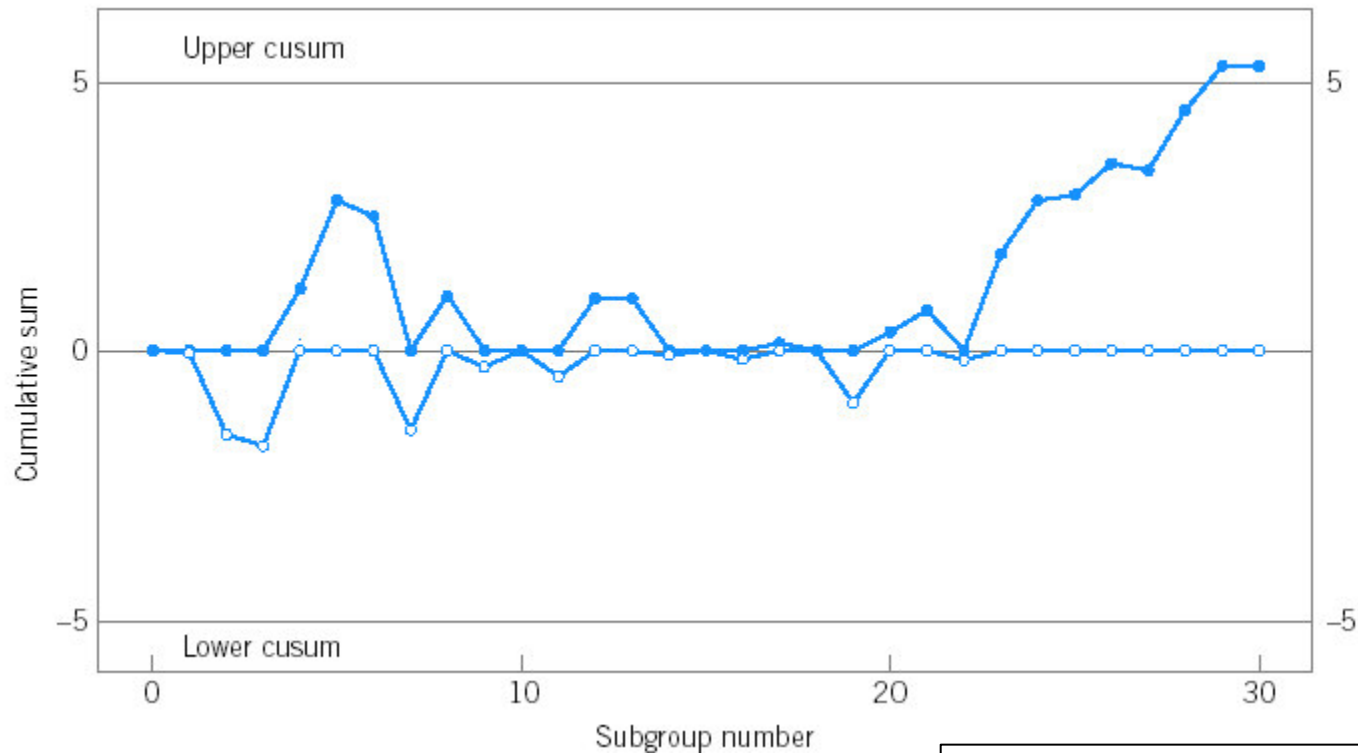
and

$$C_2^- = \max[0, 9.5 - 7.99 + 0.05] = 1.56$$

Cusum Status Chart (Figure 8-3a)



MINITAB Version of Cusum Status Chart



$$C_i^- = \min(0, x_i - \mu_0 + k + C_{i-1}^-)$$

MINITAB calculates
the lower cusum this
way

In situations where an adjustment to some manipulatable variable is required in order to bring the process back to the target value μ_0 , it may be helpful to have an estimate of the new process mean following the shift. This can be computed from

$$\hat{\mu} = \begin{cases} \mu_0 + K + \frac{C_i^+}{N^+}, & \text{if } C_i^+ > H \\ \mu_0 - K - \frac{C_i^-}{N^-}, & \text{if } C_i^- > H \end{cases} \quad (8-5)$$

To illustrate the use of equation 8-5, consider the cusum in period 29 with $C_{29}^+ = 5.28$. From equation 8-5, we would estimate the new process average as

$$\begin{aligned} \hat{\mu} &= \mu_0 + K + \frac{C_{29}^+}{N^+} \\ &= 10.0 + 0.5 + \frac{5.28}{7} \\ &= 11.25 \end{aligned}$$

Recommendations for Cusum Design

Table 8-3 ARL Performance of the Tabular Cusum with $k = \frac{1}{2}$ and $h = 4$ or $h = 5$

Shift in Mean (multiple of σ)	$h = 4$	$h = 5$
0	168	465
0.25	74.2	139
0.50	26.6	38.0
0.75	13.3	17.0
1.00	8.38	10.4
1.50	4.75	5.75
2.00	3.34	4.01
2.50	2.62	3.11
3.00	2.19	2.57
4.00	1.71	2.01

Table 8-4 Values of k and the Corresponding Values of h That Give $ARL_0 = 370$ for the Two-Sided Tabular Cusum [from Hawkins (1993a)]

k	0.25	0.5	0.75	1.0	1.25	1.5
h	8.01	4.77	3.34	2.52	1.99	1.61

The Standardized Cusum

Many users of the cusum prefer to standardize the variable x_i before performing the calculations. Let

$$y_i = \frac{x_i - \mu_0}{\sigma} \quad (8-8)$$

be the standardized value of x_i . Then the standardized cusums are defined as follows.

The Standardized Two-Sided Cusum

$$C_i^+ = \max[0, y_i - k + C_{i-1}^+] \quad (8-9)$$

$$C_i^- = \max[0, -k - y_i + C_{i-1}^-] \quad (8-10)$$

- Improving cusum performance for large shifts:
the combined cusum-Shewhart scheme

Table 8-5 ARL Values for Some Modifications of the Basic Cusum with $k = \frac{1}{2}$ and $h = 5$ (If subgroups of size $n > 1$ are used, then $\sigma = \sigma_{\bar{x}} = \sigma/\sqrt{n}$)

Shift in Mean (multiple of σ)	(a) Basic Cusum	(b) Cusum-Shewhart (Shewhart limits at 3.5σ)	(c) Cusum with FIR	(d) FIR Cusum-Shewhart (Shewhart limits at 3.5σ)
0	465	391	430	360
0.25	139	130.9	122	113.9
0.50	38.0	37.20	28.7	28.1
0.75	17.0	16.80	11.2	11.2
1.00	10.4	10.20	6.35	6.32
1.50	5.75	5.58	3.37	3.37
2.00	4.01	3.77	2.36	2.36
2.50	3.11	2.77	1.86	1.86
3.00	2.57	2.10	1.54	1.54
4.00	2.01	1.34	1.16	1.16

The Fast Initial Response (FIR) Cusum

- Set the starting value equal to $H/2$

$$C_i^+ = \max[0, x_1 - 103 + C_0^+] \\ = \max[0, 102 - 103 + 6] = 5$$

$$C_i^- = \max[0, 97 - x_1 + C_0^-] \\ = \max[0, 97 - 102 + 6] = 1$$

Table 8-6 A Cusum with a Headstart, Process Mean Equal to 100

Period i	x_i	(a)			(b)		
		$x_i - 103$	C_i^+	N^+	$97 - x_i$	C_i^-	N^-
1	102	-1	5	1	-5	1	1
2	97	-6	0	0	0	1	2
3	104	1	1	1	-7	0	0
4	93	-6	0	0	4	4	1
5	100	-3	0	0	-3	1	2
6	105	2	2	1	-8	0	0
7	96	-7	0	0	1	1	1
8	98	-5	0	0	-1	0	0
9	105	2	2	1	-8	0	0
10	99	-4	0	0	-2	0	0

Table 8-7 A Cusum with a Headstart, Process Mean Equal to 105

Period i	x_i	(a)			(b)		
		$x_i - 103$	C_i^+	N^+	$97 - x_i$	C_i^-	N^-
1	107	4	10	1	-10	0	0
2	102	-1	9	2	-5	0	0
3	109	6	15	3	-12	0	0
4	98	-5	10	4	-1	0	0
5	105	2	12	5	-8	0	0
6	110	7	19	6	-13	0	0
7	101	-2	17	7	-4	0	0
8	103	0	17	8	-6	0	0
9	110	7	24	9	-13	0	0
10	104	1	25	10	-7	0	0

- $H = 12$ implies that the cusum signals at sample 3
- Without the headstart, it would not signal until sample 6

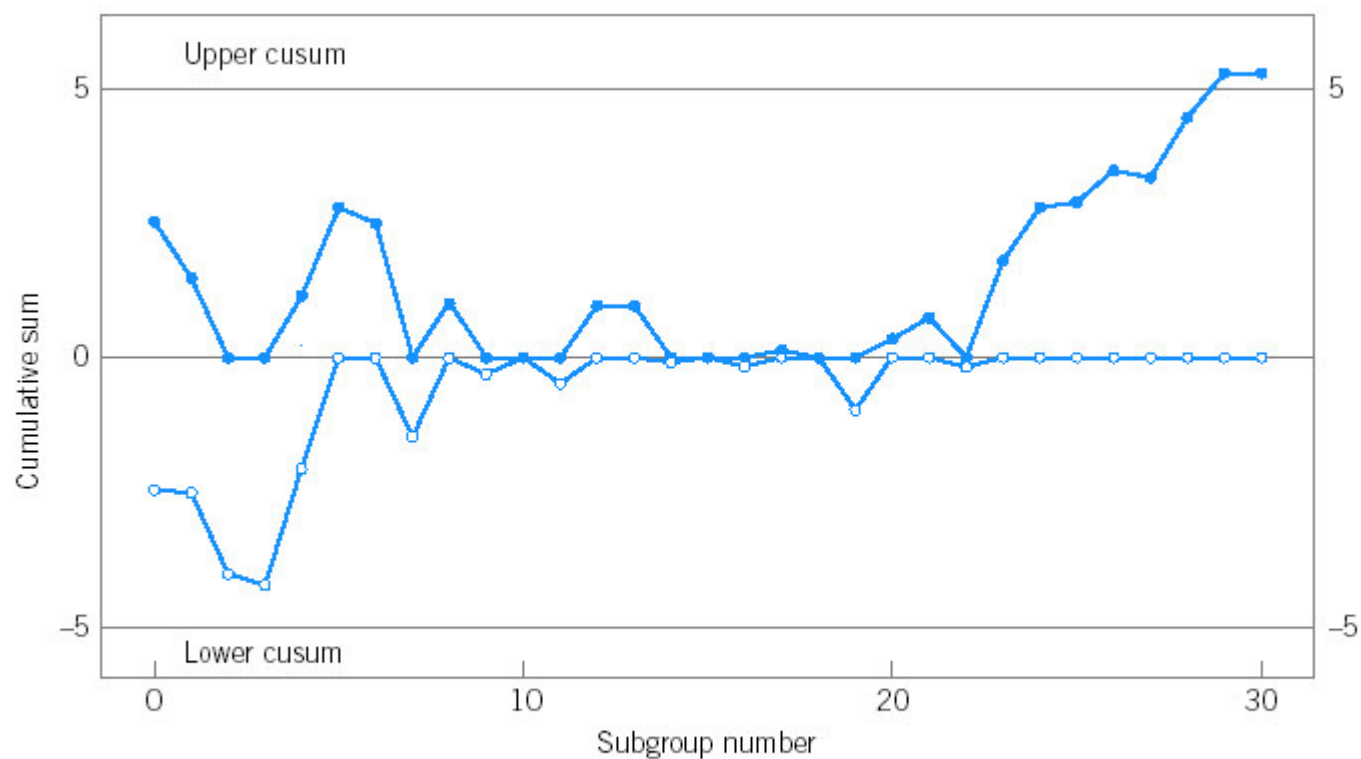


Figure 8-4 A Minitab cusum status chart for the data in Table 8-1 illustrating the fast initial response or headstart feature.

More on Cusums

- Cusums are often used to determine if a process has shifted off a specified target because it is easy to calculate the required adjustment
- One-sided cusums are often useful
- Cusums can also be used to monitor variability
- Cusums are available for other sample statistics (ranges, standard deviations, counts, proportions)
- Rational subgroups and cusums

Although we have given the development of the tabular cusum for the case of individual observations ($n = 1$), it is easily extended to the case of averages of rational subgroups where the sample size $n > 1$. Simply replace x_i by $\bar{x}_i$ (the sample or subgroup average) in the above formulas, and replace σ with $\sigma_{\bar{x}} = \sigma/\sqrt{n}$.

With Shewhart charts, the use of averages of rational subgroups substantially improves control chart performance. However, this does not always happen with the cusum. If, for example, you have a choice of taking a sample of size $n = 1$ every half hour or a sample consisting of a rational subgroup of size $n = 5$ every 2.5 hours (not that both choices have the same sampling intensity), the cusum will often work best with the choice of $n = 1$ every half hour. For more discussion of this, see Hawkins and Olwell (1998). Only if there is some significant economy of scale or some other valid reason for taking samples of size greater than unity should one consider using $n > 1$ with the cusum.

One practical reason for using rational subgroups of size $n > 1$ is that we could now set up a cusum on the sample **variance** and use it to monitor **process variability**. Cusums for variances are discussed in detail by Hawkins and Olwell (1998); the paper by Chang and Gan (1995) is also recommended.

The Cusum V-Mask

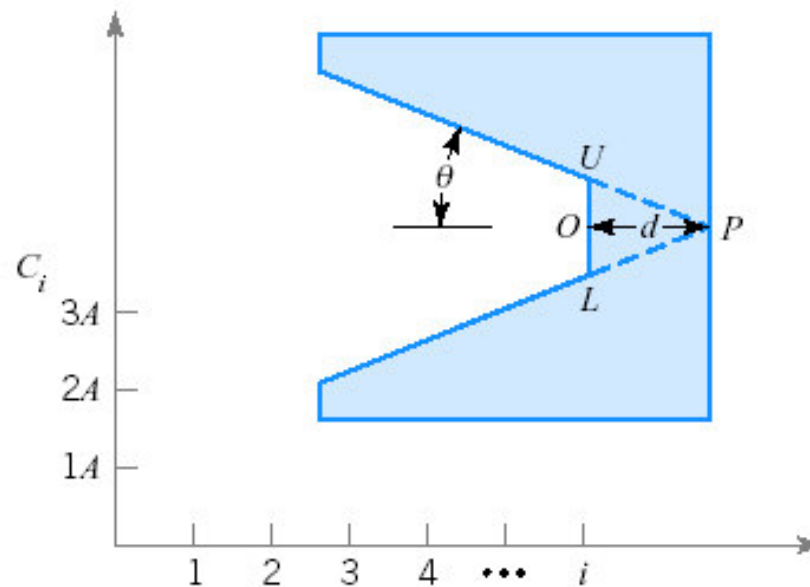


Figure 8-5 A typical V-mask.

The tabular cusum and the V-mask scheme are equivalent if

$$k = A \tan \theta \quad (8-17)$$

and

$$h = A d \tan(\theta) = dk \quad (8-18)$$

We strongly advise **against using the V-mask procedure**. Some of the disadvantages and problems associated with this scheme are as follows:

1. The headstart feature, which is very useful in practice, cannot be implemented with the V-mask.
2. It is sometimes difficult to determine how far backwards the arms of the V-mask should extend, thereby making interpretation difficult for the practitioner.
3. Perhaps the biggest problem with the V-mask is the ambiguity associated with α and β in the Johnson design procedure.

Table 8-8 Actual Values of ARL_0 for a V-Mask Scheme Designed Using Johnson's Method [Adapted from Table 2 in Woodall and Adams (1993)]

Shift to Be Detected, δ	Values of α [Desired Value of $ARL_0 = 1/(2\alpha)$]	
	0.00135 (370)	0.001 (500)
1.0	2350.6	3184.5
2.0	1804.5	2435.8
3.0	2194.8	2975.4

Adams, Lowry, and Woodall (1992) point out that defining 2α as the probability of a false alarm is incorrect. Essentially, 2α cannot be the probability of a false alarm on any single sample, because this probability changes over time on the cusum, nor can 2α be the probability of eventually obtaining a false alarm (this probability is, of course, 1). In fact, 2α must be the long-run proportion of observations resulting in false alarms. If this is so, then the in-control ARL should be $ARL_0 = 1/(2\alpha)$. However, Johnson's design method produces values of ARL_0 that are substantially larger than $1/(2\alpha)$.

8-2 THE EXPONENTIALLY WEIGHTED MOVING AVERAGE CONTROL CHART

$$z_i = \lambda x_i + (1 - \lambda)z_{i-1} \quad (8-22)$$

$$z_0 = \mu_0$$

$$\sigma_{z_i}^2 = \sigma^2 \left(\frac{\lambda}{2 - \lambda} \right) \left[1 - (1 - \lambda)^{2i} \right]$$

$$z_i = \lambda \sum_{j=0}^{i-1} (1-\lambda)^j x_{i-j} + (1-\lambda)^i z_0$$

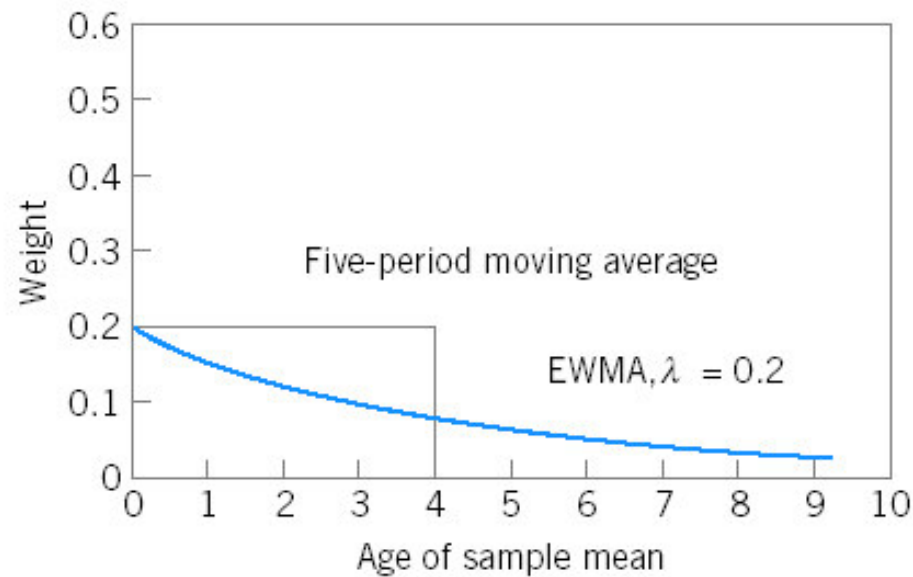


Figure 8-6 Weights of past sample means.

The EWMA Control Chart

$$UCL = \mu_0 + L\sigma \sqrt{\frac{\lambda}{(2-\lambda)} \left[1 - (1-\lambda)^{2i} \right]} \quad (8-25)$$

$$\text{Center line} = \mu_0$$

$$LCL = \mu_0 - L\sigma \sqrt{\frac{\lambda}{(2-\lambda)} \left[1 - (1-\lambda)^{2i} \right]} \quad (8-26)$$

Steady-state
control limits

$$UCL = \mu_0 + L\sigma \sqrt{\frac{\lambda}{(2-\lambda)}}$$

$$LCL = \mu_0 - L\sigma \sqrt{\frac{\lambda}{(2-\lambda)}}$$

..... EXAMPLE 8-2

We will apply the EWMA control chart with $\lambda = 0.10$ and $L = 2.7$ to the data in Table 8-1. Recall that the target value of the mean is $\mu_0 = 10$ and the standard deviation is $\sigma = 1$. The calculations for the EWMA control chart are summarized in Table 8-9, and the control chart (from Minitab) is shown in Fig. 8-7.

To illustrate the calculations, consider the first observation, $x_1 = 9.45$. The first value of the EWMA is

$$\begin{aligned} z_1 &= \lambda x_1 + (1 - \lambda)z_0 \\ &= 0.1(9.45) + 0.9(10) \\ &= 9.945 \end{aligned}$$

Therefore, $z_1 = 9.945$ is the first value plotted on the control chart in Fig. 8-7. The second value of the EWMA is

$$\begin{aligned} z_2 &= \lambda x_2 + (1 - \lambda)z_1 \\ &= 0.1(7.99) + 0.9(9.945) \\ &= 9.7495 \end{aligned}$$

Remaining calculations given in Table 8-9

Period 1 Control Limits:

$$\begin{aligned} \text{UCL} &= \mu_0 + L\sigma \sqrt{\frac{\lambda}{(2-\lambda)} \left[1 - (1-\lambda)^{2i} \right]} \\ &= 10 + 2.7(1) \sqrt{\frac{0.1}{(2-0.1)} \left[1 - (1-0.1)^{2(1)} \right]} \\ &= 10.27 \end{aligned}$$

$$\begin{aligned} \text{LCL} &= \mu_0 - L\sigma \sqrt{\frac{\lambda}{(2-\lambda)} \left[1 - (1-\lambda)^{2i} \right]} \\ &= 10 - 2.7(1) \sqrt{\frac{0.1}{(2-0.1)} \left[1 - (1-0.1)^{2(1)} \right]} \\ &= 9.73 \end{aligned}$$

Steady-State Control Limits:

$$\begin{aligned}\text{UCL} &= \mu_0 + L\sigma \sqrt{\frac{\lambda}{(2-\lambda)}} \\ &= 10 + 2.7(1) \sqrt{\frac{0.1}{(2-0.1)}} \\ &= 10.62\end{aligned}$$

$$\begin{aligned}\text{LCL} &= \mu_0 - L\sigma \sqrt{\frac{\lambda}{(2-\lambda)}} \\ &= 10 - 2.7(1) \sqrt{\frac{0.1}{(2-0.1)}} \\ &= 9.38\end{aligned}$$

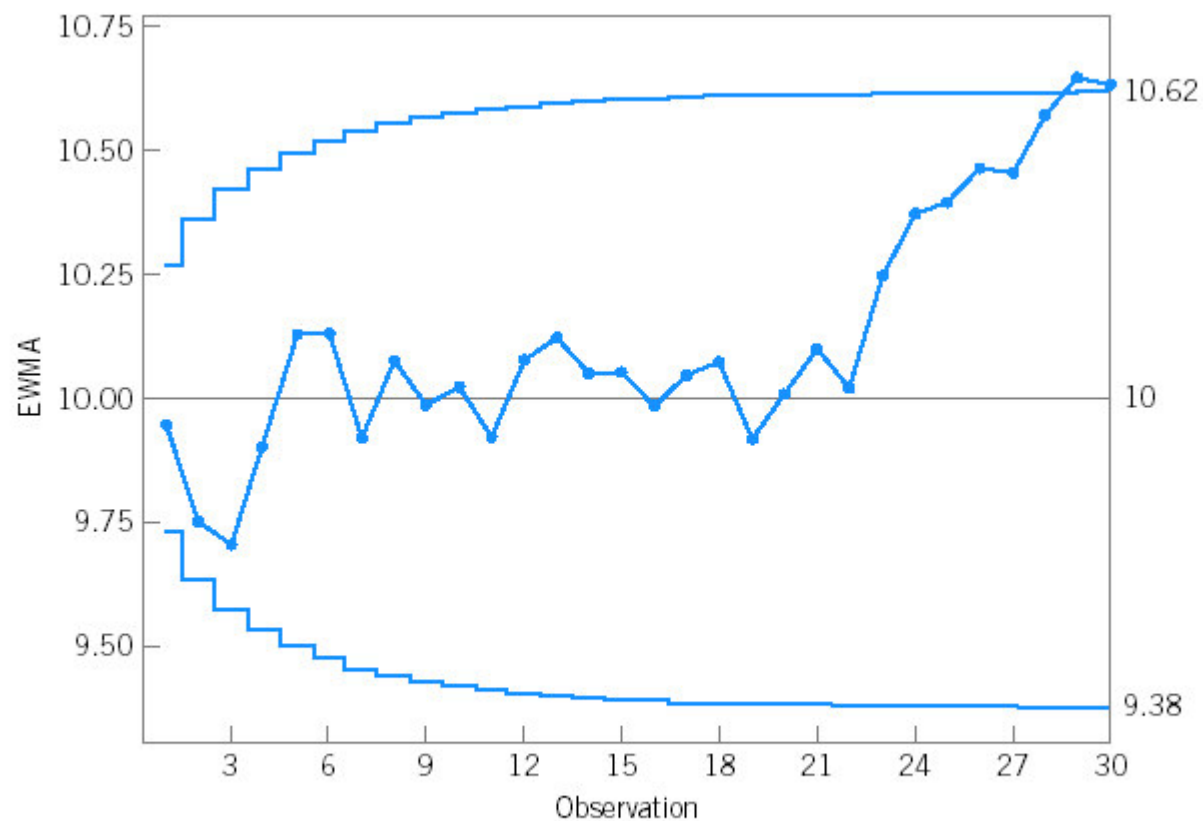


Figure 8-7 The EWMA control chart for Example 8-2.

Design of the EWMA

Table 8-10 Average Run Lengths for Several EWMA Control Schemes [Adapted from Lucas and Saccucci (1990)]

Shift in Mean (multiple of σ)	$L = 3.054$ $\lambda = 0.40$	2.998 0.25	2.962 0.20	2.814 0.10	2.615 0.05
0	500	500	500	500	500
0.25	224	170	150	106	84.1
0.50	71.2	48.2	41.8	31.3	28.8
0.75	28.4	20.1	18.2	15.9	16.4
1.00	14.3	11.1	10.5	10.3	11.4
1.50	5.9	5.5	5.5	6.1	7.1
2.00	3.5	3.6	3.7	4.4	5.2
2.50	2.5	2.7	2.9	3.4	4.2
3.00	2.0	2.3	2.4	2.9	3.5
4.00	1.4	1.7	1.9	2.2	2.7

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Shift in Mean (multiple of σ)	$L = 3.054$ $\lambda = 0.40$	2.998 0.25	2.962 0.20	2.814 0.10	2.615 0.05
0	500	500	500	500	500
0.25	224	170	150	106	84.1
0.50	71.2	48.2	41.8	31.3	28.8
0.75	28.4	20.1	18.2	15.9	16.4
1.00	14.3	11.1	10.5	10.3	11.4
1.50	5.9	5.5	5.5	6.1	7.1
2.00	3.5	3.6	3.7	4.4	5.2
2.50	2.5	2.7	2.9	3.4	4.2
3.00	2.0	2.3	2.4	2.9	3.5
4.00	1.4	1.7	1.9	2.2	2.7

Robustness of EWMA to Non-normal Process Data

Table 8-11 In-Control ARLs for the EWMA and the Individuals Control Charts for Various Gamma Distributions

λ L	EWMA			Shewhart
	0.05	0.1	0.2	1
	2.492	2.703	2.86	3.00
Normal	370.4	370.8	370.5	370.4
Gam(4, 1)	372	341	259	97
Gam(3, 1)	372	332	238	85
Gam(2, 1)	372	315	208	71
Gam(1, 1)	369	274	163	55
Gam(0.5, 1)	357	229	131	45

Table 8-12 In-Control ARLs for the EWMA and the Individuals Control Charts for Various t Distributions

λ L	EWMA			Shewhart
	0.05	0.1	0.2	1
	2.492	2.703	2.86	3.00
Normal	370.4	370.8	370.5	370.4
t_{50}	369	365	353	283
t_{40}	369	363	348	266
t_{30}	368	361	341	242
t_{20}	367	355	325	204
t_{15}	365	349	310	176
t_{10}	361	335	280	137
t_8	358	324	259	117
t_6	351	305	229	96
t_4	343	274	188	76

Rational Subgroups

The EWMA control chart is often used with individual measurements. However, if rational subgroups of size $n > 1$ are taken, then simply replace x_i with $\bar{x}_i$ and σ with $\sigma_{\bar{x}} = \sigma/\sqrt{n}$ in the previous equations.

Extensions of the EWMA

- Fast initial response feature
- Monitoring variability
- Monitoring count data
- The EWMA as a predictor of process level

8-3 THE MOVING AVERAGE CONTROL CHART

$$M_i = \frac{x_i + x_{i-1} + \cdots + x_{i-w+1}}{w}$$

$$\text{UCL} = \mu_0 + \frac{3\sigma}{\sqrt{w}}$$

$$\text{LCL} = \mu_0 - \frac{3\sigma}{\sqrt{w}}$$

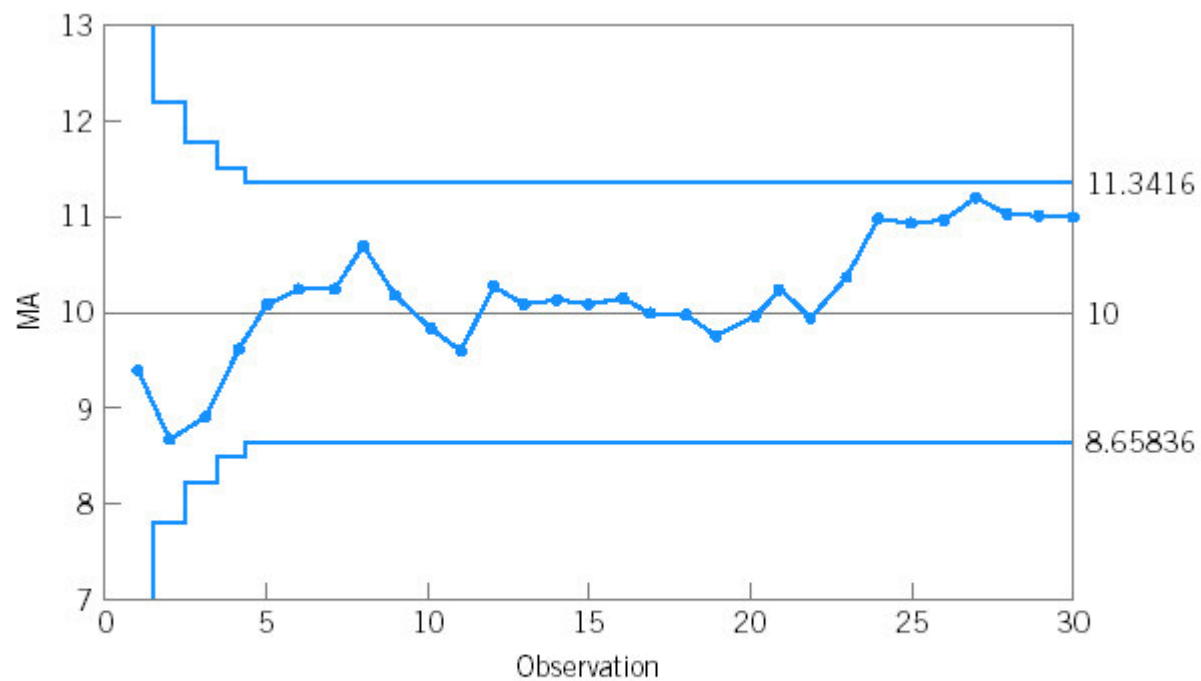


Figure 8-8 Moving average control chart with $w = 5$, Example 8-3.

IMPORTANT TERMS AND CONCEPTS

An EWMA for Poisson data	Fast initial response (headstart) feature for a cusum
ARL calculations for the cusum	Fast initial response (headstart) feature for an EWMA
Average run length	Moving average control chart
Combined cusum-Shewhart monitoring schemes	One-sided cusums
Cusum control chart	Reference value
Cusum status chart	Robustness of the EWMA to normality
Decision interval	Scale cusum
Design of a cusum	Standardized cusum
Design of an EWMA control chart	Tabular or algorithmic cusum
EWMA control chart	V-mask form of the cusum

LEARNING OBJECTIVES

1. Set up and use cusum control charts for monitoring the process mean
2. Design a cusum control chart for the mean to obtain specific ARL performance
3. Incorporate a fast initial response feature into the cusum control chart
4. Use a combined Shewhart-cusum monitoring scheme
5. Set up and use EWMA control charts for monitoring the process mean
6. Design an EWMA control chart for the mean to obtain specific ARL performance
7. Understand why the EWMA control chart is robust to the assumption of normality
8. Understand the performance advantage of cusum and EWMA control charts relative to Shewhart control charts
9. Set up and use a control chart based on an ordinary (unweighted) moving average