

# 6

## Control Charts for Attributes

---

## CHAPTER OUTLINE

---

- 6-1 INTRODUCTION
  - 6-2 THE CONTROL CHART FOR FRACTION NONCONFORMING
    - 6-2.1 Development and Operation of the Control Chart
    - 6-2.2 Variable Sample Size
    - 6-2.3 Nonmanufacturing Applications
    - 6-2.4 The Operating-Characteristic Function and Average Run Length Calculations
  - 6-3 CONTROL CHARTS FOR NONCONFORMITIES (DEFECTS)
    - 6-3.1 Procedures with Constant Sample Size
    - 6-3.2 Procedures with Variable Sample Size
    - 6-3.3 Demerit Systems
    - 6-3.4 The Operating-Characteristic Function
    - 6-3.5 Dealing with Low Defect Levels
    - 6-3.6 Nonmanufacturing Applications
  - 6-4 CHOICE BETWEEN ATTRIBUTES AND VARIABLES CONTROL CHARTS
  - 6-5 GUIDELINES FOR IMPLEMENTING CONTROL CHARTS
- Supplemental Material for Chapter 6**
- S6-1 Probability Limits on Control Charts
-

# LEARNING OBJECTIVES

1. Understand the statistical basis of attributes control charts
2. Know how to design attributes control charts
3. Know how to set up and use the  $p$  chart for fraction nonconforming
4. Know how to set up and use the  $np$  control chart for the number of nonconforming items
5. Know how to set up and use the  $c$  control chart for defects
6. Know how to set up and use the  $u$  control chart for defects per unit
7. Use attributes control charts with variable sample size
8. Understand the advantages and disadvantages of attributes versus variables control charts
9. Understand the rational subgroup concept for attributes control charts
10. Determine the average run length for attributes control charts

## 6-2 THE CONTROL CHART FOR FRACTION NONCONFORMING

The statistical principles underlying the control chart for fraction nonconforming are based on the binomial distribution. Suppose the production process is operating in a stable manner, such that the probability that any unit will not conform to specifications is  $p$ , and that successive units produced are independent. Then each unit produced is a realization of a Bernoulli random variable with parameter  $p$ . If a random sample of  $n$  units of product is selected, and if  $D$  is the number of units of product that are nonconforming, then  $D$  has a binomial distribution with parameters  $n$  and  $p$ ; that is,

$$P\{D = x\} = \binom{n}{x} p^x (1 - p)^{n-x} \quad x = 0, 1, \dots, n \quad (6-1)$$

From Section 2-2.2 we know that the mean and variance of the random variable  $D$  are  $np$  and  $np(1 - p)$ , respectively.

The **sample fraction nonconforming** is defined as the ratio of the number of nonconforming units in the sample  $D$  to the sample size  $n$ ; that is,

$$\hat{p} = \frac{D}{n} \quad (6-2)$$

As noted in Section 2-2.2, the distribution of the random variable  $\hat{p}$  can be obtained from the binomial. Furthermore, the mean and variance of  $\hat{p}$  are

$$\mu = p \quad (6-3)$$

and

$$\sigma_{\hat{p}}^2 = \frac{p(1-p)}{n} \quad (6-4)$$

respectively.

If  $w$  is a statistic that measures a quality characteristic, and if the mean of  $w$  is  $\mu_w$  and the variance of  $w$  is  $\sigma_w^2$ , then the general model for the Shewhart control chart is as follows:

$$\begin{aligned} \text{UCL} &= \mu_w + L\sigma_w \\ \text{Center line} &= \mu_w \\ \text{LCL} &= \mu_w - L\sigma_w \end{aligned} \tag{6-5}$$

**Fraction Nonconforming Control Chart: Standard Given**

$$\begin{aligned} \text{UCL} &= p + 3\sqrt{\frac{p(1-p)}{n}} \\ \text{Center line} &= p \\ \text{LCL} &= p - 3\sqrt{\frac{p(1-p)}{n}} \end{aligned} \tag{6-6}$$

When the process fraction nonconforming  $p$  is not known, then it must be estimated from observed data. The usual procedure is to select  $m$  preliminary samples, each of size  $n$ . As a general rule,  $m$  should be 20 or 25. Then if there are  $D_i$  nonconforming units in sample  $i$ , we compute the fraction nonconforming in the  $i$ th sample as

$$\hat{p}_i = \frac{D_i}{n} \quad i = 1, 2, \dots, m$$

and the average of these individual sample fractions nonconforming is

$$\bar{p} = \frac{\sum_{i=1}^m D_i}{mn} = \frac{\sum_{i=1}^m \hat{p}_i}{m} \quad (6-7)$$

The statistic  $\bar{p}$  estimates the unknown fraction nonconforming  $p$ . The center line and control limits of the control chart for fraction nonconforming are computed as follows:

**Fraction Nonconforming Control Chart: No Standard Given**

$$\begin{aligned} \text{UCL} &= \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \\ \text{Center line} &= \bar{p} \\ \text{LCL} &= \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \end{aligned} \tag{6-8}$$

## ..... EXAMPLE 6-1 .....

Frozen orange juice concentrate is packed in 6-oz cardboard cans. These cans are formed on a machine by spinning them from cardboard stock and attaching a metal bottom panel. By inspection of a can, we may determine whether, when filled, it could possibly leak either on the side seam or around the bottom joint. Such a nonconforming can has an improper seal on either the side seam or the bottom panel. We wish to set up a control chart to improve the fraction of nonconforming cans produced by this machine.

To establish the control chart, 30 samples of  $n = 50$  cans each were selected at half-hour intervals over a three-shift period in which the machine was in continuous operation. The data are shown in Table 6-1.

**Table 6-1** Data for Trial Control Limits, Example 6-1, Sample Size  $n = 50$

| Sample Number | Number of Nonconforming Cans, $D_i$ | Sample Fraction Nonconforming, $\hat{p}_i$ | Sample Number | Number of Nonconforming Cans, $D_i$ | Sample Fraction Nonconforming, $\hat{p}_i$ |
|---------------|-------------------------------------|--------------------------------------------|---------------|-------------------------------------|--------------------------------------------|
| 1             | 12                                  | 0.24                                       | 17            | 10                                  | 0.20                                       |
| 2             | 15                                  | 0.30                                       | 18            | 5                                   | 0.10                                       |
| 3             | 8                                   | 0.16                                       | 19            | 13                                  | 0.26                                       |
| 4             | 10                                  | 0.20                                       | 20            | 11                                  | 0.22                                       |
| 5             | 4                                   | 0.08                                       | 21            | 20                                  | 0.40                                       |
| 6             | 7                                   | 0.14                                       | 22            | 18                                  | 0.36                                       |
| 7             | 16                                  | 0.32                                       | 23            | 24                                  | 0.48                                       |
| 8             | 9                                   | 0.18                                       | 24            | 15                                  | 0.30                                       |
| 9             | 14                                  | 0.28                                       | 25            | 9                                   | 0.18                                       |
| 10            | 10                                  | 0.20                                       | 26            | 12                                  | 0.24                                       |
| 11            | 5                                   | 0.10                                       | 27            | 7                                   | 0.14                                       |
| 12            | 6                                   | 0.12                                       | 28            | 13                                  | 0.26                                       |
| 13            | 17                                  | 0.34                                       | 29            | 9                                   | 0.18                                       |
| 14            | 12                                  | 0.24                                       | 30            | 6                                   | 0.12                                       |
| 15            | 22                                  | 0.44                                       |               | 347                                 | $\bar{p} = 0.2313$                         |
| 16            | 8                                   | 0.16                                       |               |                                     |                                            |

We construct a phase I control chart using this preliminary data to determine if the process was in control when these data were collected. Since the 30 samples contain  $\sum_{i=1}^{30} D_i = 347$  nonconforming cans, we find from equation 6-7,

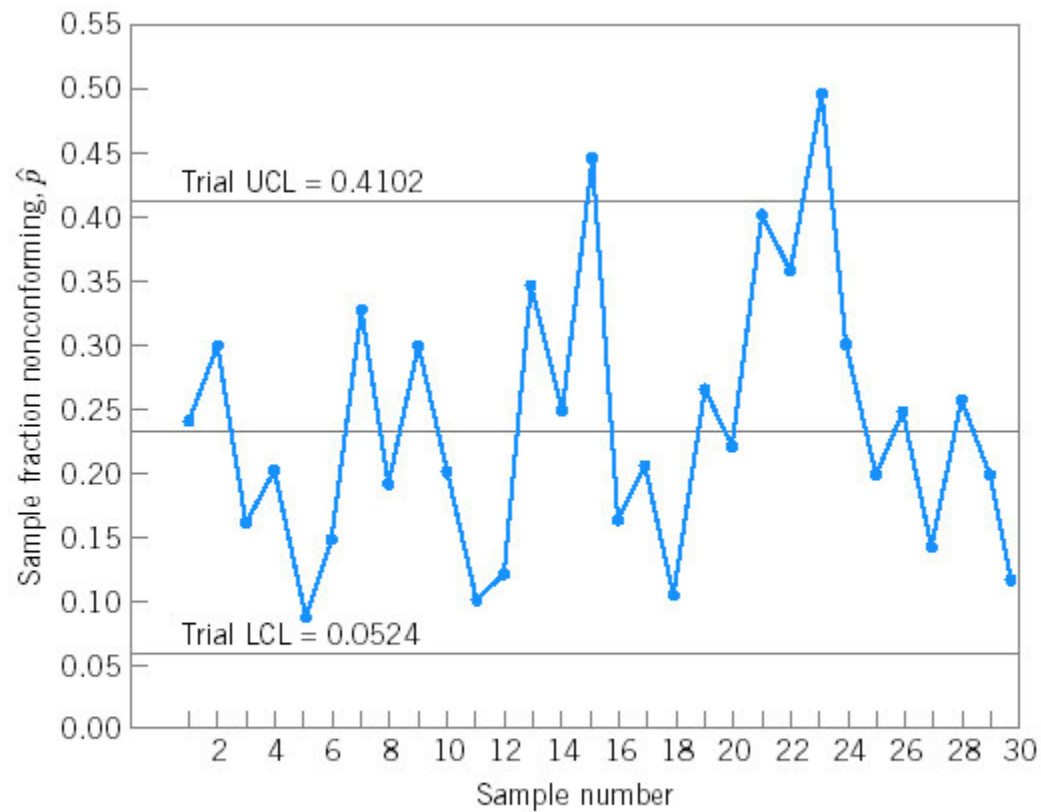
$$\bar{p} = \frac{\sum_{i=1}^m D_i}{mn} = \frac{347}{(30)(50)} = 0.2313$$

Using  $\bar{p}$  as an estimate of the true process fraction nonconforming, we can now calculate the upper and lower control limits as

$$\begin{aligned}\bar{p} \pm 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} &= 0.2313 \pm 3\sqrt{\frac{0.2313(0.7687)}{50}} \\ &= 0.2313 \pm 3(0.0596) \\ &= 0.2313 \pm 0.1789\end{aligned}$$

$$\text{UCL} = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.2313 + 0.1789 = 0.4102$$

$$\text{LCL} = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.2313 - 0.1789 = 0.0524$$



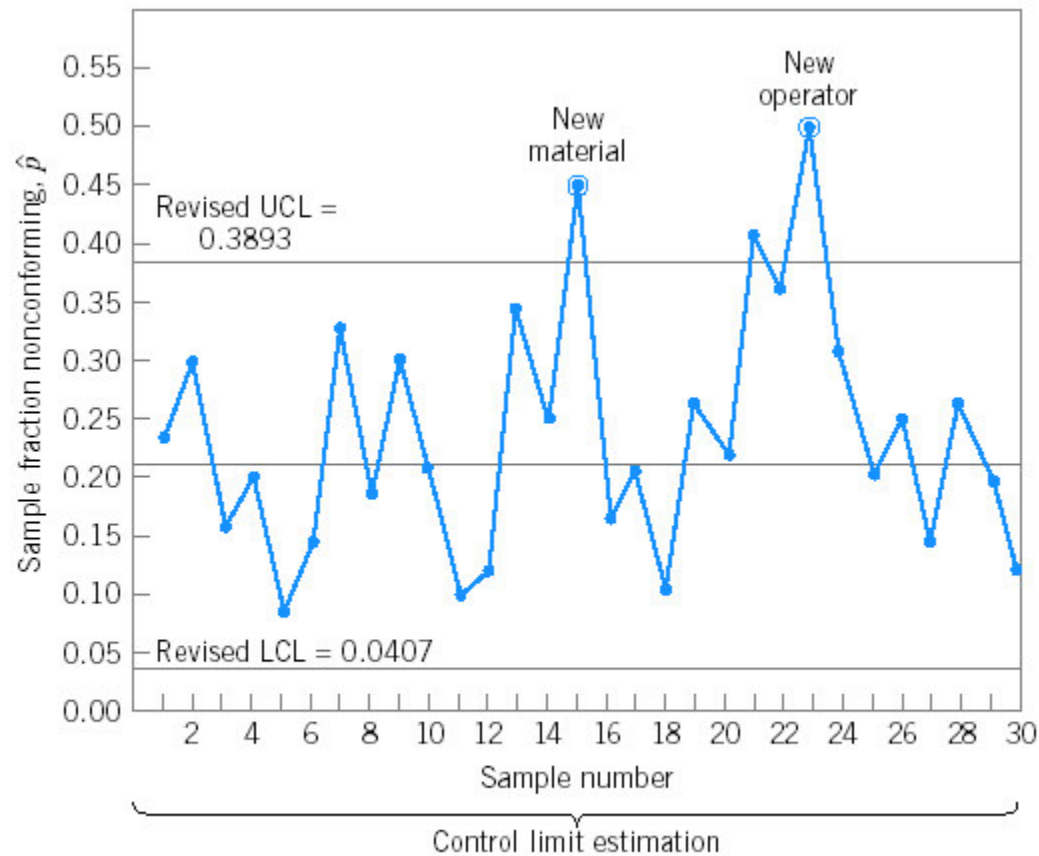
**Figure 6-1** Initial phase I fraction nonconforming control chart for the data in Table 6-1.

samples 15 and 23 are eliminated, and the new center line and revised control limits are calculated as

$$\bar{p} = \frac{301}{(28)(50)} = 0.2150$$

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.2150 + 3\sqrt{\frac{0.2150(0.7850)}{50}} = 0.3893$$

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.2150 - 3\sqrt{\frac{0.2150(0.7850)}{50}} = 0.0407$$

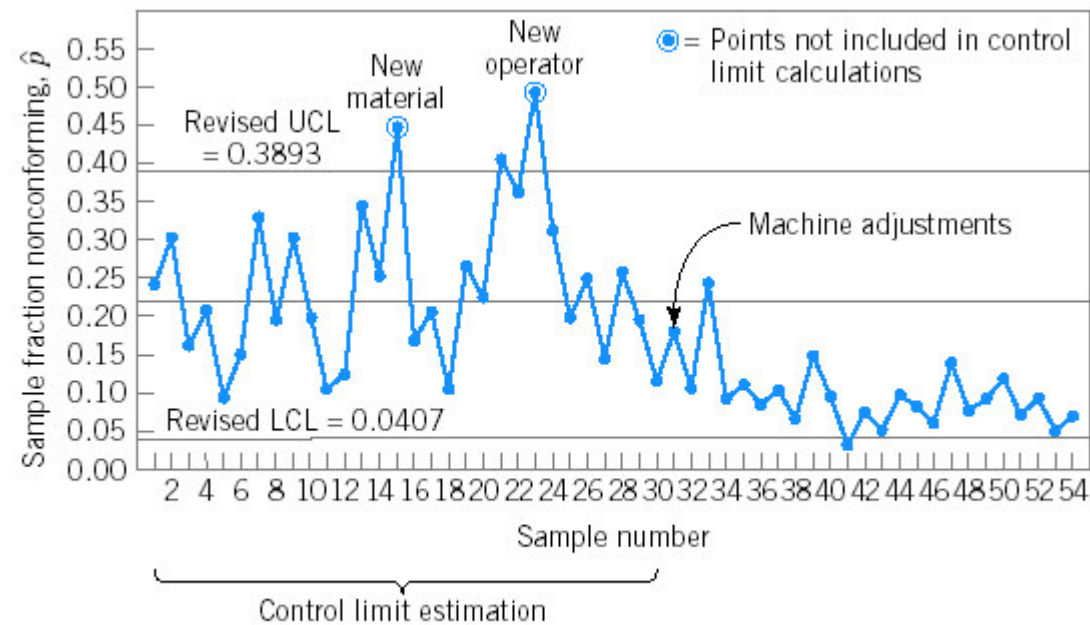


**Figure 6-2** Revised control limits for the data in Table 6-1.

During the next three shifts following the machine adjustments and the introduction of the control chart, an additional 24 samples of  $n = 50$  observations each are collected. These data are shown in Table 6-2, and the sample fractions nonconforming are plotted on the control chart in Fig. 6-3.

**Table 6-2** Orange Juice Concentrate Can Data in Samples of Size  $n = 50$

| Sample Number | Number of Nonconforming Cans, $D_i$ | Sample Fraction Nonconforming, $\hat{p}_i$ | Sample Number | Number of Nonconforming Cans, $D_i$ | Sample Fraction Nonconforming, $\hat{p}_i$ |
|---------------|-------------------------------------|--------------------------------------------|---------------|-------------------------------------|--------------------------------------------|
| 31            | 9                                   | 0.18                                       | 44            | 6                                   | 0.12                                       |
| 32            | 6                                   | 0.12                                       | 45            | 5                                   | 0.10                                       |
| 33            | 12                                  | 0.24                                       | 46            | 4                                   | 0.08                                       |
| 34            | 5                                   | 0.10                                       | 47            | 8                                   | 0.16                                       |
| 35            | 6                                   | 0.12                                       | 48            | 5                                   | 0.10                                       |
| 36            | 4                                   | 0.08                                       | 49            | 6                                   | 0.12                                       |
| 37            | 6                                   | 0.12                                       | 50            | 7                                   | 0.14                                       |
| 38            | 3                                   | 0.06                                       | 51            | 5                                   | 0.10                                       |
| 39            | 7                                   | 0.14                                       | 52            | 6                                   | 0.12                                       |
| 40            | 6                                   | 0.12                                       | 53            | 3                                   | 0.06                                       |
| 41            | 2                                   | 0.04                                       | 54            | 5                                   | 0.10                                       |
| 42            | 4                                   | 0.08                                       |               | 133                                 | $\bar{p} = 0.1108$                         |
| 43            | 3                                   | 0.06                                       |               |                                     |                                            |



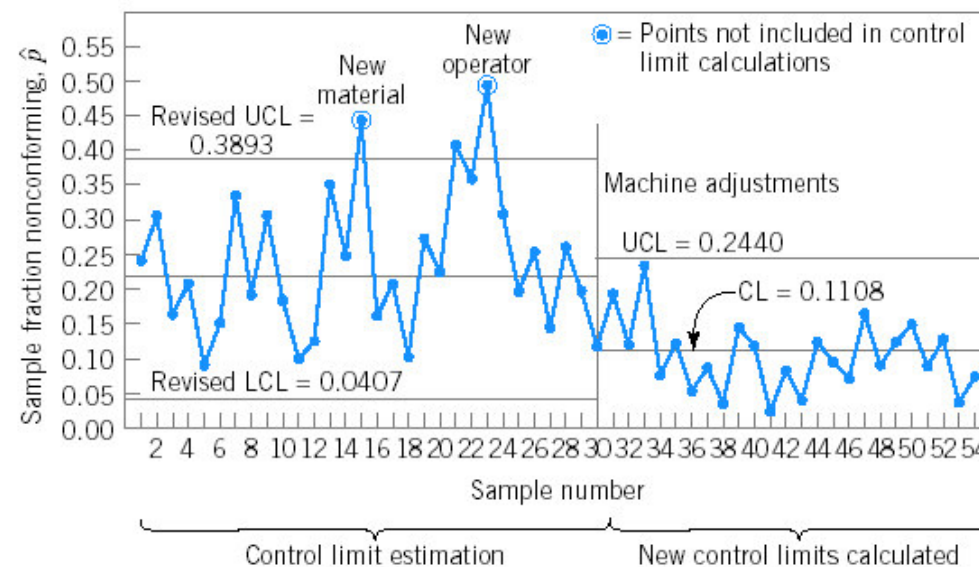
**Figure 6-3** Continuation of the fraction nonconforming control chart, Example 6-1.

Based on the apparently successful process adjustments, it seems logical to revise the control limits again, using only the most recent samples (numbers 31–54). This results in the new control chart parameters:

$$\text{Center line} = \bar{p} = 0.1108$$

$$\text{UCL} = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.1108 + 3\sqrt{\frac{(0.1108)(0.8892)}{50}} = 0.2440$$

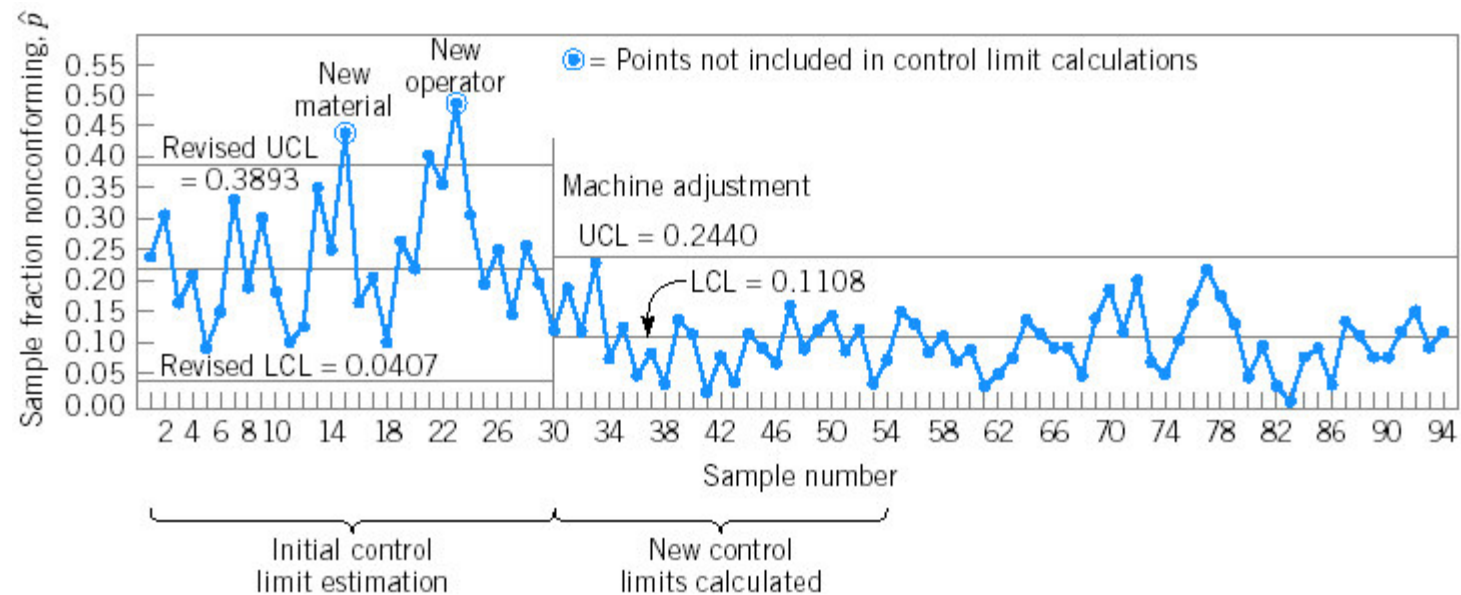
$$\text{LCL} = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.1108 - 3\sqrt{\frac{(0.1108)(0.8892)}{50}} = -0.0224 = 0$$



**Figure 6-4** New control limits on the fraction nonconforming control chart, Example 6-1.

**Table 6-3** New Data for the Fraction Nonconforming Control Chart in Fig. 6-5,  $n = 50$ 

| Sample Number | Number of Nonconforming Cans, $D_i$ | Sample Fraction Nonconforming, $\hat{p}_i$ | Sample Number | Number of Nonconforming Cans, $D_i$ | Sample Fraction Nonconforming, $\hat{p}_i$ |
|---------------|-------------------------------------|--------------------------------------------|---------------|-------------------------------------|--------------------------------------------|
| 55            | 8                                   | 0.16                                       | 75            | 5                                   | 0.10                                       |
| 56            | 7                                   | 0.14                                       | 76            | 8                                   | 0.16                                       |
| 57            | 5                                   | 0.10                                       | 77            | 11                                  | 0.22                                       |
| 58            | 6                                   | 0.12                                       | 78            | 9                                   | 0.18                                       |
| 59            | 4                                   | 0.08                                       | 79            | 7                                   | 0.14                                       |
| 60            | 5                                   | 0.10                                       | 80            | 3                                   | 0.06                                       |
| 61            | 2                                   | 0.04                                       | 81            | 5                                   | 0.10                                       |
| 62            | 3                                   | 0.06                                       | 82            | 2                                   | 0.04                                       |
| 63            | 4                                   | 0.08                                       | 83            | 1                                   | 0.02                                       |
| 64            | 7                                   | 0.14                                       | 84            | 4                                   | 0.08                                       |
| 65            | 6                                   | 0.12                                       | 85            | 5                                   | 0.10                                       |
| 66            | 5                                   | 0.10                                       | 86            | 3                                   | 0.06                                       |
| 67            | 5                                   | 0.10                                       | 87            | 7                                   | 0.14                                       |
| 68            | 3                                   | 0.06                                       | 88            | 6                                   | 0.12                                       |
| 69            | 7                                   | 0.14                                       | 89            | 4                                   | 0.08                                       |
| 70            | 9                                   | 0.18                                       | 90            | 4                                   | 0.08                                       |
| 71            | 6                                   | 0.12                                       | 91            | 6                                   | 0.12                                       |
| 72            | 10                                  | 0.20                                       | 92            | 8                                   | 0.16                                       |
| 73            | 4                                   | 0.08                                       | 93            | 5                                   | 0.10                                       |
| 74            | 3                                   | 0.06                                       | 94            | 6                                   | 0.12                                       |



**Figure 6-5** Completed fraction nonconforming control chart, Example 6-1.

# Design of Fraction Nonconforming Chart

- Three parameters must be specified
  1. The sample size
  2. The frequency of sampling
  3. The width of the control limits
- Common to base chart on 100% inspection of *all* process output over time
- Rational subgroups may also play role in determining sampling frequency

# Interpretation of Points on the Control Chart for Fraction Nonconforming

Example 6-1 illustrated how points that plot beyond the control limits are treated, both in establishing the control chart and during its routine operation. Care must be exercised in interpreting points that plot *below* the lower control limit. These points often do not represent a real improvement in process quality. Frequently, they are caused by errors in the inspection process resulting from inadequately trained or inexperienced inspectors or from improperly calibrated test and inspection equipment. We have also seen cases in which inspectors deliberately passed nonconforming units or reported fictitious data. The analyst must keep these warnings in mind when looking for assignable causes if points plot below the lower control limits. Not all “downward shifts” in  $p$  are attributable to improved quality.

# The $np$ Control Chart

It is also possible to base a control chart on the number nonconforming rather than the fraction nonconforming. This is often called an  $np$  control chart. The parameters of this chart are as follows.

## The $np$ Control Chart

$$\begin{aligned} \text{UCL} &= np + 3\sqrt{np(1-p)} \\ \text{Center line} &= np \\ \text{LCL} &= np - 3\sqrt{np(1-p)} \end{aligned} \tag{6-13}$$

## 6-2.2 Variable Sample Size

### Variable-Width Control Limits

The first and perhaps the most simple approach is to determine control limits for each individual sample that are based on the specific sample size. That is, if the  $i$ th sample is of size  $n_i$ , then the upper and lower control limits are  $\bar{p} \pm 3\sqrt{\bar{p}(1-\bar{p})/n_i}$ .

**Table 6-4** Data for a Control Chart for Fraction Nonconforming with Variable Sample Size

| Sample<br>Number, $i$ | Sample<br>Size, $n_i$ | Number of<br>Nonconforming<br>Units, $D_i$ | Sample Fraction<br>Nonconforming,<br>$\hat{p}_i = D_i/n_i$ | Standard Deviation                                           | Control Limits<br>LCL | Limits<br>UCL |
|-----------------------|-----------------------|--------------------------------------------|------------------------------------------------------------|--------------------------------------------------------------|-----------------------|---------------|
|                       |                       |                                            |                                                            | $\hat{\sigma}_{\hat{p}} = \sqrt{\frac{(0.096)(0.904)}{n_i}}$ |                       |               |
| 1                     | 100                   | 12                                         | 0.120                                                      | 0.029                                                        | 0.009                 | 0.183         |
| 2                     | 80                    | 8                                          | 0.100                                                      | 0.033                                                        | 0                     | 0.195         |
| 3                     | 80                    | 6                                          | 0.075                                                      | 0.033                                                        | 0                     | 0.195         |
| 4                     | 100                   | 9                                          | 0.090                                                      | 0.029                                                        | 0.009                 | 0.183         |
| 5                     | 110                   | 10                                         | 0.091                                                      | 0.028                                                        | 0.012                 | 0.180         |
| 6                     | 110                   | 12                                         | 0.109                                                      | 0.028                                                        | 0.012                 | 0.180         |
| 7                     | 100                   | 11                                         | 0.110                                                      | 0.029                                                        | 0.009                 | 0.183         |
| 8                     | 100                   | 16                                         | 0.160                                                      | 0.029                                                        | 0.009                 | 0.183         |
| 9                     | 90                    | 10                                         | 0.110                                                      | 0.031                                                        | 0.003                 | 0.189         |
| 10                    | 90                    | 6                                          | 0.067                                                      | 0.031                                                        | 0.003                 | 0.189         |
| 11                    | 110                   | 20                                         | 0.182                                                      | 0.028                                                        | 0.012                 | 0.180         |
| 12                    | 120                   | 15                                         | 0.125                                                      | 0.027                                                        | 0.015                 | 0.177         |
| 13                    | 120                   | 9                                          | 0.075                                                      | 0.027                                                        | 0.015                 | 0.177         |
| 14                    | 120                   | 8                                          | 0.067                                                      | 0.027                                                        | 0.015                 | 0.177         |
| 15                    | 110                   | 6                                          | 0.055                                                      | 0.028                                                        | 0.012                 | 0.180         |
| 16                    | 80                    | 8                                          | 0.100                                                      | 0.033                                                        | 0                     | 0.195         |
| 17                    | 80                    | 10                                         | 0.125                                                      | 0.033                                                        | 0                     | 0.195         |
| 18                    | 80                    | 7                                          | 0.088                                                      | 0.033                                                        | 0                     | 0.195         |
| 19                    | 90                    | 5                                          | 0.056                                                      | 0.031                                                        | 0.003                 | 0.189         |
| 20                    | 100                   | 8                                          | 0.080                                                      | 0.029                                                        | 0.009                 | 0.183         |
| 21                    | 100                   | 5                                          | 0.050                                                      | 0.029                                                        | 0.009                 | 0.183         |
| 22                    | 100                   | 8                                          | 0.080                                                      | 0.029                                                        | 0.009                 | 0.183         |
| 23                    | 100                   | 10                                         | 0.100                                                      | 0.029                                                        | 0.009                 | 0.183         |
| 24                    | 90                    | 6                                          | 0.067                                                      | 0.031                                                        | 0.003                 | 0.189         |
| 25                    | 90                    | 9                                          | 0.100                                                      | 0.031                                                        | 0.003                 | 0.189         |
|                       | 2450                  | 234                                        | 2.383                                                      |                                                              |                       |               |

To illustrate this approach, consider the data in Table 6-4. For the 25 samples, we calculate

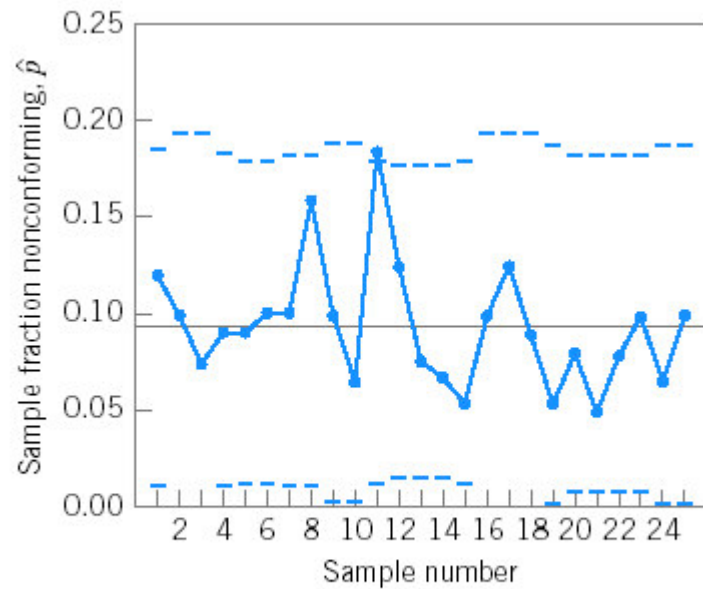
$$\bar{p} = \frac{\sum_{i=1}^{25} D_i}{\sum_{i=1}^{25} n_i} = \frac{234}{2450} = 0.096$$

Consequently, the center line is at 0.096, and the control limits are

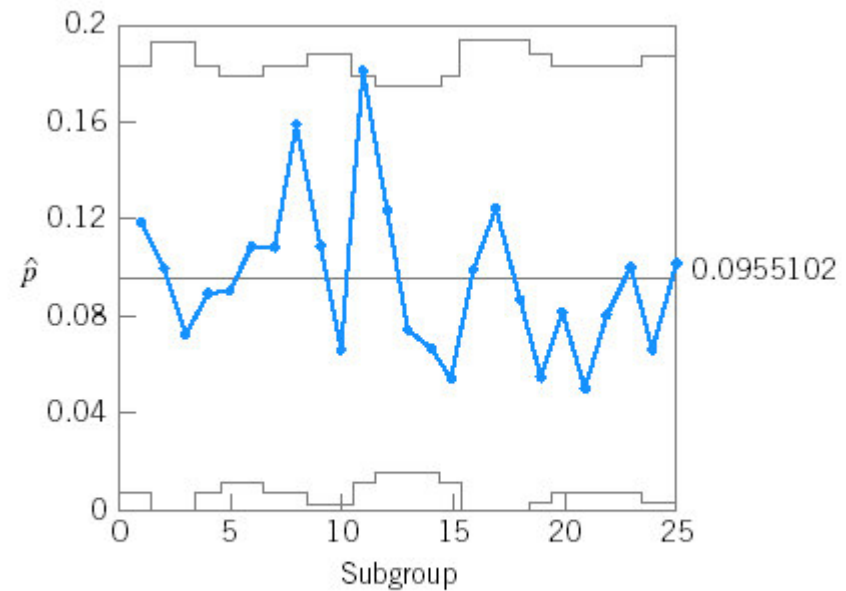
$$UCL = \bar{p} + 3\hat{\sigma}_{\hat{p}} = 0.096 + 3\sqrt{\frac{(0.096)(0.904)}{n_i}}$$

and

$$LCL = \bar{p} - 3\hat{\sigma}_{\hat{p}} = 0.096 - 3\sqrt{\frac{(0.096)(0.904)}{n_i}}$$



**Figure 6-6** Control chart for fraction nonconforming with variable sample size.



**Figure 6-7** Control chart for fraction nonconforming with variable sample size using Minitab.

## 6-2.2 Variable Sample Size

### Control Limits Based on an Average Sample Size

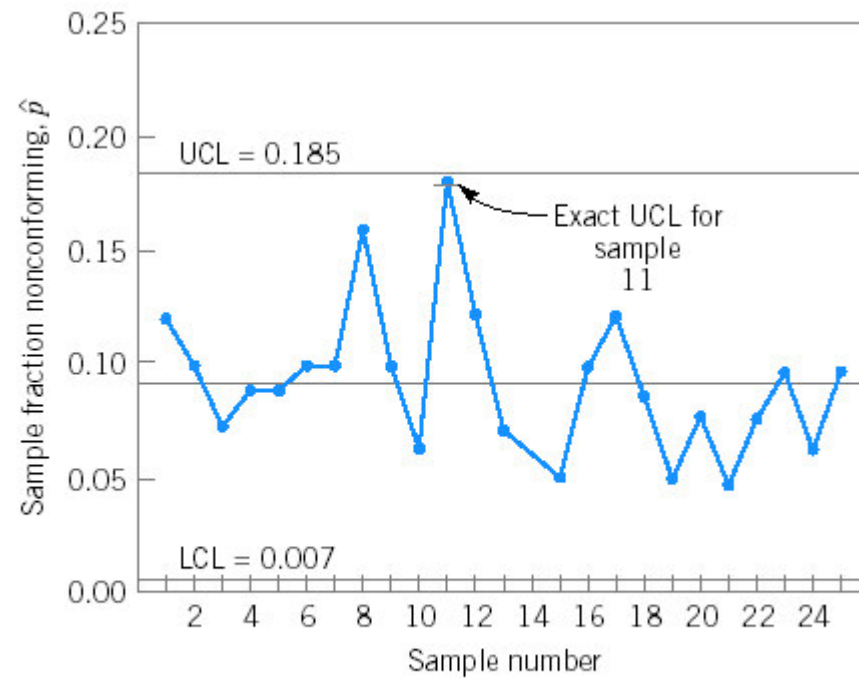
The second approach is to base the control chart on an *average sample size*, resulting in an **approximate set of control limits**. This assumes that future sample sizes will not differ greatly from those previously observed.

For the data in Table 6-4, we find that the average sample size is

$$\bar{n} = \frac{\sum_{i=1}^{25} n_i}{25} = \frac{2450}{25} = 98$$

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{\bar{n}}} = 0.096 + 3\sqrt{\frac{(0.096)(0.904)}{98}} = 0.185$$

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{\bar{n}}} = 0.096 - 3\sqrt{\frac{(0.096)(0.904)}{98}} = 0.007$$



**Figure 6-8** Control chart for fraction nonconforming based on average sample size.

## 6-2.2 Variable Sample Size

### The Standardized Control Chart

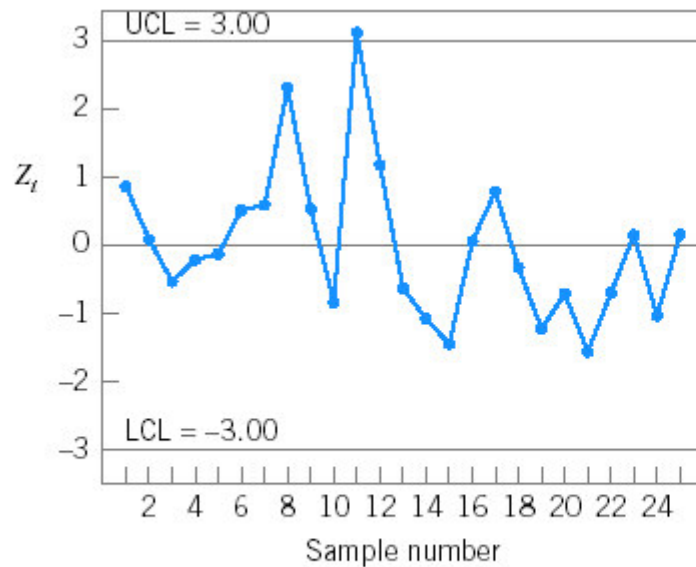
The third approach to dealing with variable sample size is to use a “standardized” control chart, where the points are plotted in standard deviation units. Such a control chart has the center line at zero, and upper and lower control limits of +3 and −3, respectively. The variable plotted on the chart is

$$Z_i = \frac{\hat{p}_i - p}{\sqrt{\frac{p(1-p)}{n_i}}} \quad (6-14)$$

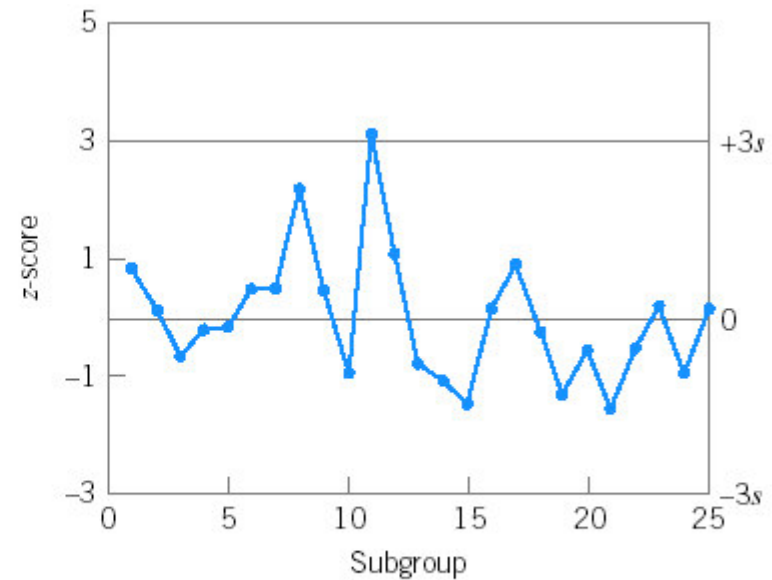
where  $p$  (or  $\bar{p}$  if no standard is given) is the process fraction nonconforming in the in-control state. The standardized control chart for the data in Table 6-4 is shown in Fig. 6-9.

**Table 6-5** Calculations for the Standardized Control Chart in Fig. 6-9,  $\bar{p} = 0.096$

| Sample<br>Number, $i$ | Sample<br>Size, $n_i$ | Number of<br>Noncon-<br>forming<br>Units, $D_i$ | Sample<br>Fraction<br>Noncon-<br>forming,<br>$\hat{p}_i = D_i/n_i$ | Standard Deviation<br>$\hat{\sigma}_p = \sqrt{\frac{(0.096)(0.904)}{n_i}}$ | $Z_i = \frac{\hat{p}_i - \bar{p}}{\sqrt{\frac{(0.096)(0.904)}{n_i}}}$ |
|-----------------------|-----------------------|-------------------------------------------------|--------------------------------------------------------------------|----------------------------------------------------------------------------|-----------------------------------------------------------------------|
| 1                     | 100                   | 12                                              | 0.120                                                              | 0.029                                                                      | 0.83                                                                  |
| 2                     | 80                    | 8                                               | 0.100                                                              | 0.033                                                                      | 0.12                                                                  |
| 3                     | 80                    | 6                                               | 0.075                                                              | 0.033                                                                      | -0.64                                                                 |
| 4                     | 100                   | 9                                               | 0.090                                                              | 0.029                                                                      | -0.21                                                                 |
| 5                     | 110                   | 10                                              | 0.091                                                              | 0.028                                                                      | -0.18                                                                 |
| 6                     | 110                   | 12                                              | 0.109                                                              | 0.028                                                                      | 0.46                                                                  |
| 7                     | 100                   | 11                                              | 0.110                                                              | 0.029                                                                      | 0.48                                                                  |
| 8                     | 100                   | 16                                              | 0.160                                                              | 0.029                                                                      | 2.21                                                                  |
| 9                     | 90                    | 10                                              | 0.110                                                              | 0.031                                                                      | 0.45                                                                  |
| 10                    | 90                    | 6                                               | 0.067                                                              | 0.031                                                                      | -0.94                                                                 |
| 11                    | 110                   | 20                                              | 0.182                                                              | 0.028                                                                      | 3.07                                                                  |
| 12                    | 120                   | 15                                              | 0.125                                                              | 0.027                                                                      | 1.07                                                                  |
| 13                    | 120                   | 9                                               | 0.075                                                              | 0.027                                                                      | -0.78                                                                 |
| 14                    | 120                   | 8                                               | 0.067                                                              | 0.027                                                                      | -1.07                                                                 |
| 15                    | 110                   | 6                                               | 0.055                                                              | 0.028                                                                      | -1.46                                                                 |
| 16                    | 80                    | 8                                               | 0.100                                                              | 0.033                                                                      | 0.12                                                                  |
| 17                    | 80                    | 10                                              | 0.125                                                              | 0.033                                                                      | 0.88                                                                  |
| 18                    | 80                    | 7                                               | 0.088                                                              | 0.033                                                                      | -0.24                                                                 |
| 19                    | 90                    | 5                                               | 0.056                                                              | 0.031                                                                      | -1.29                                                                 |
| 20                    | 100                   | 8                                               | 0.080                                                              | 0.029                                                                      | -0.55                                                                 |
| 21                    | 100                   | 5                                               | 0.050                                                              | 0.029                                                                      | -1.59                                                                 |
| 22                    | 100                   | 8                                               | 0.080                                                              | 0.029                                                                      | -0.55                                                                 |
| 23                    | 100                   | 10                                              | 0.100                                                              | 0.029                                                                      | 0.14                                                                  |
| 24                    | 90                    | 6                                               | 0.067                                                              | 0.031                                                                      | -0.94                                                                 |
| 25                    | 90                    | 9                                               | 0.100                                                              | 0.031                                                                      | 0.13                                                                  |



**Figure 6-9** Standardized control chart for fraction nonconforming.



**Figure 6-10** Standardized control chart from Minitab for fraction nonconforming, Table 6-4.

## 6-2.4 The Operating-Characteristic Function and Average Run Length Calculations

The probability of type II error for the fraction nonconforming control chart may be computed from

$$\begin{aligned}\beta &= P\{\hat{p} < \text{UCL}|p\} - P\{\hat{p} \leq \text{LCL}|p\} \\ &= P\{D < n\text{UCL}|p\} - P\{D \leq \text{LCL}|p\}\end{aligned}\quad (6-15)$$

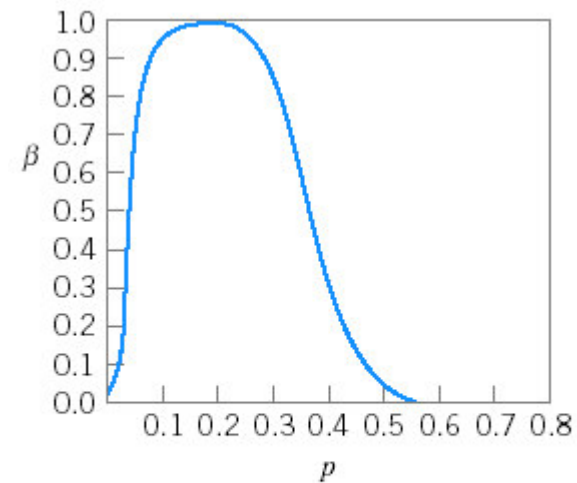
Table 6-6 illustrates the calculations required to generate the OC curve for a control chart for fraction nonconforming with parameters  $n = 50$ ,  $LCL = 0.0303$ , and  $UCL = 0.3697$ . Using these parameters, equation 6-15 becomes

$$\begin{aligned}\beta &= P\{D < (50)(0.3697)|p\} - P\{D \leq (50)(0.0303)|p\} \\ &= P\{D < 18.49|p\} - P\{D \leq 1.52|p\}\end{aligned}$$

However, since  $D$  must be an integer, we find that

$$\beta = P\{D \leq 18|p\} - P\{D \leq 1|p\}$$

The OC curve is plotted in Fig. 6-11.



**Figure 6-11** Operating-characteristic curve for the fraction nonconforming control chart with  $\bar{p} = 0.20$ ,  $LCL = 0.0303$ , and  $UCL = 0.3697$ .

**Table 6-6** Calculations<sup>a</sup> for Constructing the OC Curve for a Control Chart for Fraction Nonconforming with  $n = 50$ , LCL = 0.0303, and UCL = 0.3697

| $p$  | $P\{D \leq 18   p\}$ | $P\{D \leq 1   p\}$ | $\beta = P\{D \leq 18   p\} - P\{D \leq 1   p\}$ |
|------|----------------------|---------------------|--------------------------------------------------|
| 0.01 | 1.0000               | 0.9106              | 0.0894                                           |
| 0.03 | 1.0000               | 0.5553              | 0.4447                                           |
| 0.05 | 1.0000               | 0.2794              | 0.7206                                           |
| 0.10 | 1.0000               | 0.0338              | 0.9662                                           |
| 0.15 | 0.9999               | 0.0029              | 0.9970                                           |
| 0.20 | 0.9975               | 0.0002              | 0.9973                                           |
| 0.25 | 0.9713               | 0.0000              | 0.9713                                           |
| 0.30 | 0.8594               | 0.0000              | 0.8594                                           |
| 0.35 | 0.6216               | 0.0000              | 0.6216                                           |
| 0.40 | 0.3356               | 0.0000              | 0.3356                                           |
| 0.45 | 0.1273               | 0.0000              | 0.1273                                           |
| 0.50 | 0.0325               | 0.0000              | 0.0325                                           |
| 0.55 | 0.0053               | 0.0000              | 0.0053                                           |

<sup>a</sup>The probabilities in this table were found by evaluating the cumulative binomial distribution. For small  $p$  ( $p < 0.1$ , say) the Poisson approximation could be used, and for larger values of  $p$  the normal approximation could be used.

We may also calculate average run lengths (ARLs) for the fraction nonconforming control chart. Recall from Chapter 4 that for uncorrelated process data the ARL for any Shewhart control chart can be written as

$$ARL = \frac{1}{P(\text{sample point plots out of control})}$$

Thus, if the process is in control,  $ARL_0$  is

$$ARL_0 = \frac{1}{\alpha}$$

and if it is out of control, then

$$ARL_1 = \frac{1}{1 - \beta}$$

To illustrate, consider the control chart for fraction nonconforming used in the OC curve calculations in Table 6-6. This chart has parameters  $n = 50$ ,  $UCL = 0.3697$ ,  $LCL = 0.0303$ , and the center line is  $\bar{p} = 0.20$ . From Table 6-6 (or the OC curve in Fig. 6-11) we find that if the process is in control with  $p = \bar{p}$ , the probability of a point plotting in control is 0.9973. Thus, in this case  $\alpha = 1 - \beta = 0.0027$ , and the value of  $ARL_0$  is

$$ARL_0 = \frac{1}{\alpha} = \frac{1}{0.0027} \simeq 370$$

Therefore, if the process is really in control, we will experience a “false-alarm” out-of-control signal about every 370 samples. (This will be approximately true, in general, for any Shewhart control chart with three-sigma limits.) This in-control  $ARL_0$  is generally considered to be satisfactorily large. Now suppose that the process shifts out of control to  $p = 0.3$ . Table 6-6 indicates that if  $p = 0.3$ , then  $\beta = 0.8594$ . Therefore, the value of  $ARL_1$  is

$$ARL_1 = \frac{1}{1 - \beta} = \frac{1}{1 - 0.8594} \simeq 7$$

and it will take about seven samples, on the average, to detect this shift with a point outside of the control limits.

## 6-3 CONTROL CHARTS FOR NONCONFORMITIES (DEFECTS)

### 6-3.1 Procedures with Constant Sample Size

Consider the occurrence of nonconformities in an inspection unit of product. In most cases, the inspection unit will be a single unit of product, although this is not necessarily always so. The inspection unit is simply an entity for which it is convenient to keep records. It could be a group of 5 units of product, 10 units of product, and so on. Suppose that defects or nonconformities occur in this inspection unit according to the Poisson distribution; that is,

$$p(x) = \frac{e^{-c} c^x}{x!} \quad x = 0, 1, 2, \dots$$

where  $x$  is the number of nonconformities and  $c > 0$  is the parameter of the Poisson distribution.

Therefore, a control chart for nonconformities with three-sigma limits<sup>2</sup> would be defined as follows,

**Control Chart for Nonconformities: Standard Given**

$$\begin{aligned} \text{UCL} &= c + 3\sqrt{c} \\ \text{Center line} &= c \\ \text{LCL} &= c - 3\sqrt{c} \end{aligned} \tag{6-16}$$

---

<sup>2</sup>The  $\alpha$ -risk for three-sigma limits is not equally allocated above the UCL and below the LCL, because the Poisson distribution is asymmetric. Some authors recommend the use of probability limits for this chart, particularly when  $c$  is small.

assuming that a standard value for  $c$  is available. Should these calculations yield a negative value for the LCL, set  $\text{LCL} = 0$ .

If no standard is given, then  $c$  may be estimated as the observed average number of nonconformities in a preliminary sample of inspection units—say,  $\bar{c}$ . In this case, the control chart has parameters defined as follows.

**Control Chart for Nonconformities: No Standard Given**

$$\begin{aligned} \text{UCL} &= \bar{c} + 3\sqrt{\bar{c}} \\ \text{Center line} &= \bar{c} \\ \text{LCL} &= \bar{c} - 3\sqrt{\bar{c}} \end{aligned} \tag{6-17}$$

When no standard is given, the control limits in equation 6-17 should be regarded as *trial* control limits, and the preliminary samples examined for lack of control in the usual phase I analysis. The control chart for nonconformities is also sometimes called the  $c$  chart.

..... **EXAMPLE 6-3** .....

Table 6-7 presents the number of nonconformities observed in 26 successive samples of 100 printed circuit boards. Note that, for reasons of convenience, the inspection unit is defined as 100 boards.

**Table 6-7** Data on the Number of Nonconformities in Samples of 100 Printed Circuit Boards

| Sample Number | Number of<br>Nonconformities | Sample Number | Number of<br>Nonconformities |
|---------------|------------------------------|---------------|------------------------------|
| 1             | 21                           | 14            | 19                           |
| 2             | 24                           | 15            | 10                           |
| 3             | 16                           | 16            | 17                           |
| 4             | 12                           | 17            | 13                           |
| 5             | 15                           | 18            | 22                           |
| 6             | 5                            | 19            | 18                           |
| 7             | 28                           | 20            | 39                           |
| 8             | 20                           | 21            | 30                           |
| 9             | 31                           | 22            | 24                           |
| 10            | 25                           | 23            | 16                           |
| 11            | 20                           | 24            | 19                           |
| 12            | 24                           | 25            | 17                           |
| 13            | 16                           | 26            | 15                           |

Since the 26 samples contain 516 total nonconformities, we estimate  $c$  by

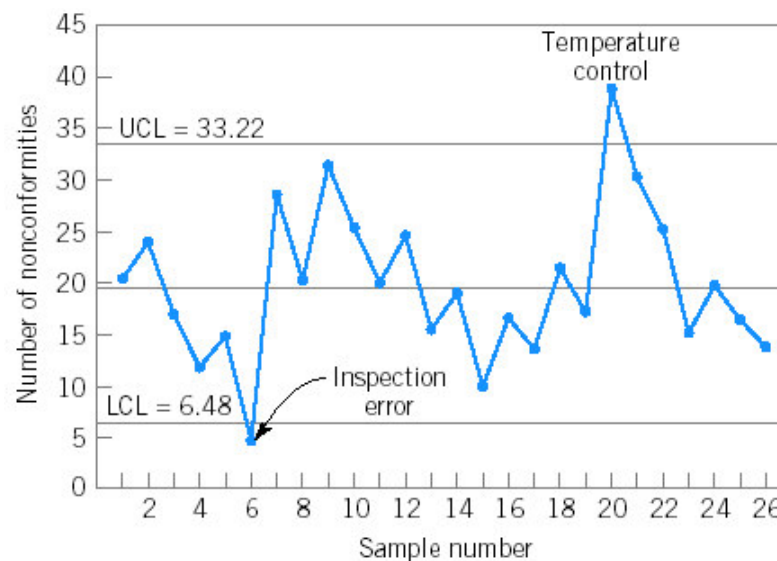
$$\bar{c} = \frac{516}{26} = 19.85$$

Therefore, the trial control limits are given by

$$UCL = \bar{c} + 3\sqrt{\bar{c}} = 19.85 + 3\sqrt{19.85} = 33.22$$

$$\text{Center line} = \bar{c} = 19.85$$

$$LCL = \bar{c} - 3\sqrt{\bar{c}} = 19.85 - 3\sqrt{19.85} = 6.48$$



**Figure 6-12** Control chart for nonconformities for Example 6-3.

The control chart is shown in Fig. 6-12. The number of observed nonconformities from the preliminary samples is plotted on this chart. Two points plot outside the control limits, samples 6 and 20. Investigation of sample 6 revealed that a new inspector had examined the boards in this sample and that he did not recognize several of the types of nonconformities that could have been present. Furthermore, the unusually large number of nonconformities in sample 20 resulted from a temperature control problem in the wave soldering machine, which was subsequently repaired. Therefore, it seems reasonable to exclude these two samples and revise the trial control limits. The estimate of  $c$  is now computed as

$$\bar{c} = \frac{472}{24} = 19.67$$

and the revised control limits are

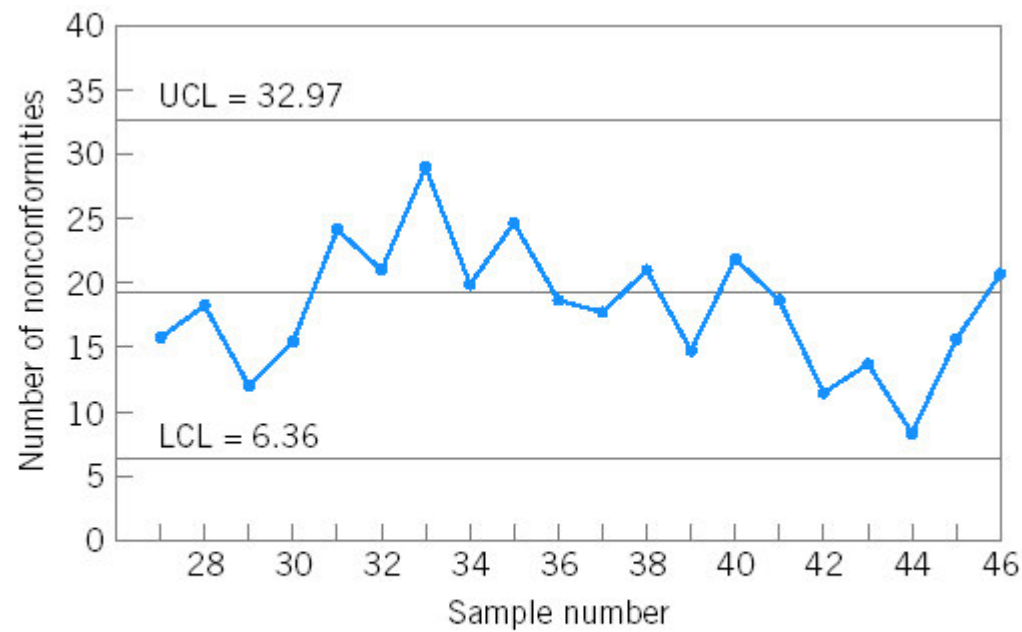
$$\begin{aligned} \text{UCL} &= \bar{c} + 3\sqrt{\bar{c}} = 19.67 + 3\sqrt{19.67} = 32.97 \\ \text{Center line} &= \bar{c} = 19.67 \\ \text{LCL} &= \bar{c} - 3\sqrt{\bar{c}} = 19.67 - 3\sqrt{19.67} = 6.36 \end{aligned}$$

These become the standard values against which production in the next period can be compared.

Twenty new samples, each consisting of one inspection unit (i.e., 100 boards), are subsequently collected. The number of nonconformities in each sample is noted and recorded in Table 6-8. These points are plotted on the control chart in Fig. 6-13. No lack of control is indicated; however, the number of nonconformities per board is still unacceptably high. Management action is necessary to improve the process.

**Table 6-8** Additional Data for the Control Chart for Nonconformities, Example 6-3

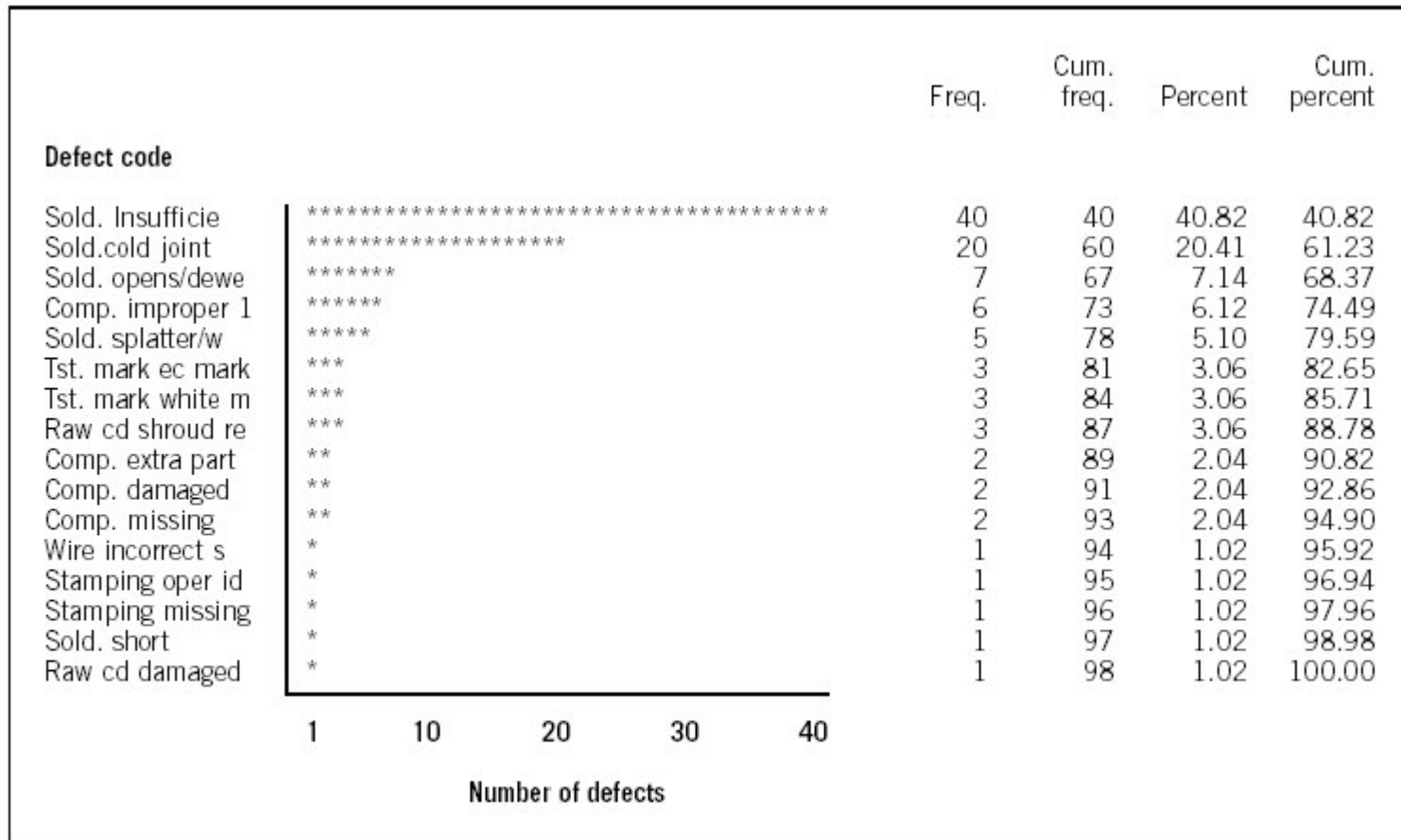
| Sample Number | Number of<br>Nonconformities | Sample Number | Number of<br>Nonconformities |
|---------------|------------------------------|---------------|------------------------------|
| 27            | 16                           | 37            | 18                           |
| 28            | 18                           | 38            | 21                           |
| 29            | 12                           | 39            | 16                           |
| 30            | 15                           | 40            | 22                           |
| 31            | 24                           | 41            | 19                           |
| 32            | 21                           | 42            | 12                           |
| 33            | 28                           | 43            | 14                           |
| 34            | 20                           | 44            | 9                            |
| 35            | 25                           | 45            | 16                           |
| 36            | 19                           | 46            | 21                           |



**Figure 6-13** Continuation of the control chart for nonconformities, Example 6-3.



# Further Analysis of Nonconformities

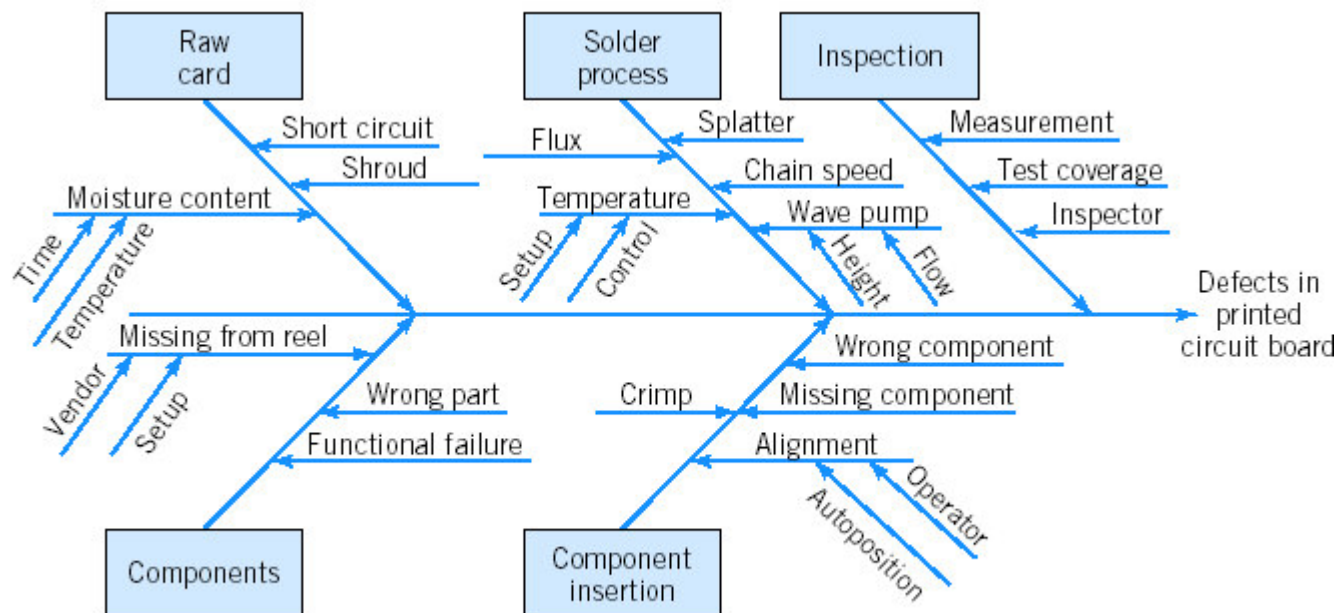


**Figure 6-14** Pareto analysis of nonconformities for the printed circuit board process.

- Refer to Table 6-9 for occurrence of defect type by type of printed circuit board (part number)

Another useful technique for further analysis of nonconformities is the **cause-and-effect diagram** discussed in Chapter 4. The cause-and-effect diagram is used to illustrate the various sources of nonconformities in products and their interrelationships. It is useful in focusing the attention of operators, manufacturing engineers, and managers on quality problems. Developing a good cause-and-effect diagram usually advances the level of technological understanding of the process.

A cause-and-effect diagram for the printed circuit board assembly process is shown in Fig. 6-15. Since most of the defects in this example were solder related, the cause-and-effect diagram could be used here to help choose the variables for a designed experiment to optimize the wave soldering process.



**Figure 6-15** Cause-and-effect diagram.

# Choice of Sample Size: The $u$ Chart

The second approach involves setting up a control chart based on the average number of nonconformities per inspection unit. If we find  $x$  *total* nonconformities in a sample of  $n$  inspection units, then the *average* number of nonconformities per inspection unit is

$$u = \frac{x}{n} \quad (6-18)$$

## Control Chart for Average Number of Nonconformities per Unit

$$\begin{aligned} \text{UCL} &= \bar{u} + 3\sqrt{\frac{\bar{u}}{n}} \\ \text{Center line} &= \bar{u} \\ \text{LCL} &= \bar{u} - 3\sqrt{\frac{\bar{u}}{n}} \end{aligned} \quad (6-19)$$

### ..... EXAMPLE 6-4 .....

A supply chain engineering group monitors shipments of materials through the company distribution network. Errors on either the delivered material or the accompanying documentation are tracked on a weekly basis. Fifty randomly selected shipments are examined and the errors recorded. Data for 20 weeks are shown in Table 6-10.

**Table 6-10** Data on Number of Shipping Errors in a Supply Chain Network

| Sample<br>Number (week), $i$ | Sample<br>Size, $n$ | Total Number of Errors<br>(Nonconformities), $x_i$ | Average Number of<br>Errors (Nonconformities)<br>per Unit, $u_i = x_i/n$ |
|------------------------------|---------------------|----------------------------------------------------|--------------------------------------------------------------------------|
| 1                            | 50                  | 2                                                  | 0.04                                                                     |
| 2                            | 50                  | 3                                                  | 0.06                                                                     |
| 3                            | 50                  | 8                                                  | 0.16                                                                     |
| 4                            | 50                  | 1                                                  | 0.02                                                                     |
| 5                            | 50                  | 1                                                  | 0.02                                                                     |
| 6                            | 50                  | 4                                                  | 0.08                                                                     |
| 7                            | 50                  | 1                                                  | 0.02                                                                     |
| 8                            | 50                  | 4                                                  | 0.08                                                                     |
| 9                            | 50                  | 5                                                  | 0.10                                                                     |
| 10                           | 50                  | 1                                                  | 0.02                                                                     |
| 11                           | 50                  | 8                                                  | 0.16                                                                     |
| 12                           | 50                  | 2                                                  | 0.04                                                                     |
| 13                           | 50                  | 4                                                  | 0.08                                                                     |
| 14                           | 50                  | 3                                                  | 0.06                                                                     |
| 15                           | 50                  | 4                                                  | 0.08                                                                     |
| 16                           | 50                  | 1                                                  | 0.02                                                                     |
| 17                           | 50                  | 8                                                  | 0.16                                                                     |
| 18                           | 50                  | 3                                                  | 0.06                                                                     |
| 19                           | 50                  | 7                                                  | 0.14                                                                     |
| 20                           | 50                  | 4                                                  | 0.08                                                                     |
|                              |                     | <hr/> 74                                           | <hr/> 1.48                                                               |

A  $u$  control chart can be established to monitor this process. From these data, we estimate the number of errors (nonconformities) per unit (shipment) to be:

$$\bar{u} = \frac{\sum_{i=1}^{20} u_i}{20} = \frac{1.48}{20} = 0.0740$$

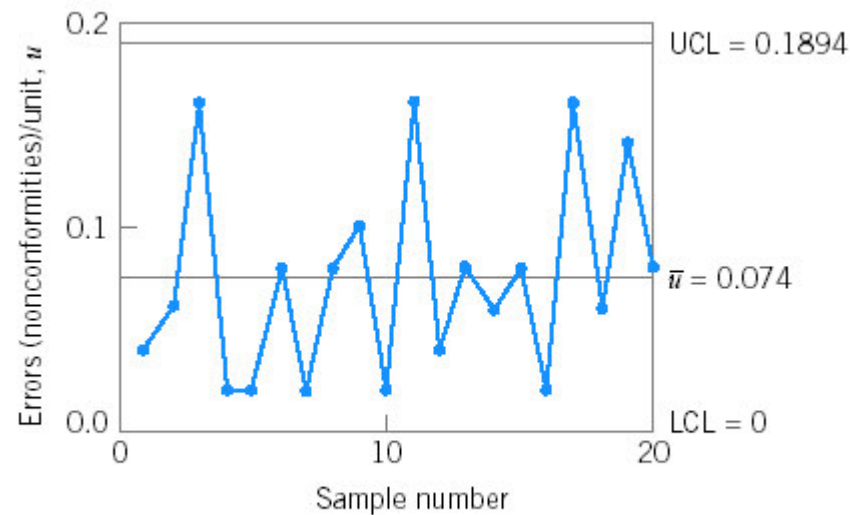
Therefore, the parameters of the control chart are

$$\text{UCL} = \bar{u} + 3\sqrt{\frac{\bar{u}}{n}} = 0.0740 + 3\sqrt{\frac{0.0740}{50}} = 0.1894$$

$$\text{Center line} = \bar{u} = 0.0740$$

$$\text{LCL} = \bar{u} - 3\sqrt{\frac{\bar{u}}{n}} = 0.0740 - 3\sqrt{\frac{0.0740}{50}} = -0.0414$$

Since the  $\text{LCL} < 0$ , we would set  $\text{LCL} = 0$  for the  $u$  chart. The control chart is shown in Fig. 6-16.



**Figure 6-16** The control chart for nonconformities per unit from Minitab for Example 6-4.

The preliminary data do not exhibit lack of statistical control; therefore, the trial control limits given here would be adopted for phase II monitoring of future operations. Once again, note that, although the process is in control, the average number of errors per shipment is high. Action should be taken to improve the supply chain system.

## 6-3 CONTROL CHARTS FOR NONCONFORMITIES (DEFECTS)

### 6-3.2 Procedures with Variable Sample Size

#### ..... EXAMPLE 6-5 .....

In a textile finishing plant, dyed cloth is inspected for the occurrence of defects per 50 square meters. The data on ten rolls of cloth are shown in Table 6-11. We will use these data to set up a control chart for nonconformities per unit.

**Table 6-11** Occurrence of Nonconformities in Dyed Cloth

| Roll Number | Number of Square Meters | Total Number of Nonconformities | Number of Inspection Units in Roll, $n$ | Number of Nonconformities per Inspection Unit |
|-------------|-------------------------|---------------------------------|-----------------------------------------|-----------------------------------------------|
| 1           | 500                     | 14                              | 10.0                                    | 1.40                                          |
| 2           | 400                     | 12                              | 8.0                                     | 1.50                                          |
| 3           | 650                     | 20                              | 13.0                                    | 1.54                                          |
| 4           | 500                     | 11                              | 10.0                                    | 1.10                                          |
| 5           | 475                     | 7                               | 9.5                                     | 0.74                                          |
| 6           | 500                     | 10                              | 10.0                                    | 1.00                                          |
| 7           | 600                     | 21                              | 12.0                                    | 1.75                                          |
| 8           | 525                     | 16                              | 10.5                                    | 1.52                                          |
| 9           | 600                     | 19                              | 12.0                                    | 1.58                                          |
| 10          | 625                     | 23                              | 12.5                                    | 1.84                                          |
|             |                         | <u>153</u>                      | <u>107.50</u>                           |                                               |

The center line of the chart should be the average number of nonconformities per inspection unit—that is, the average number of nonconformities per 50 square meters, computed as

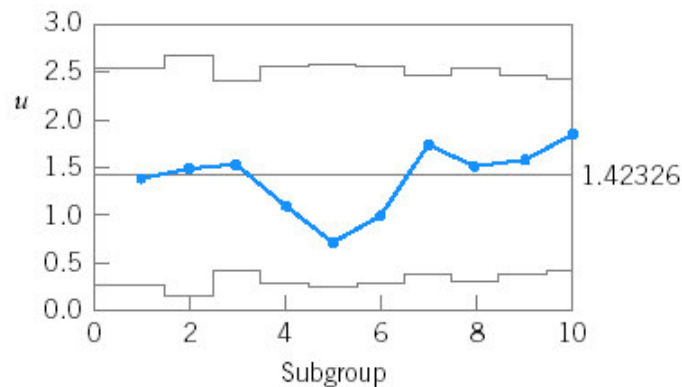
$$\bar{u} = \frac{153}{107.5} = 1.42$$

Note that  $\bar{u}$  is the ratio of the total number of observed nonconformities to the total number of inspection units.

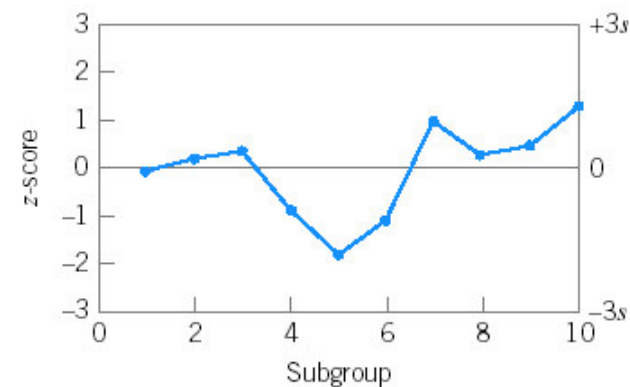
The control limits on this chart are computed from equation 6-19 with  $n$  replaced by  $n_i$ . The width of the control limits will vary inversely with  $n_i$ , the number of inspection units in the roll. The calculations for the control limits are displayed in Table 6-12. Figure 6-17 plots the control chart constructed by Minitab.

**Table 6-12** Calculation of Control Limits, Example 6-5

| Roll<br>Number, $i$ | $n_i$ | $UCL = \bar{u} + 3\sqrt{\bar{u}/n_i}$ | $LCL = \bar{u} - 3\sqrt{\bar{u}/n_i}$ |
|---------------------|-------|---------------------------------------|---------------------------------------|
| 1                   | 10.0  | 2.55                                  | 0.29                                  |
| 2                   | 8.0   | 2.68                                  | 0.16                                  |
| 3                   | 13.0  | 2.41                                  | 0.43                                  |
| 4                   | 10.0  | 2.55                                  | 0.29                                  |
| 5                   | 9.5   | 2.58                                  | 0.26                                  |
| 6                   | 10.0  | 2.55                                  | 0.29                                  |
| 7                   | 12.0  | 2.45                                  | 0.39                                  |
| 8                   | 10.5  | 2.52                                  | 0.32                                  |
| 9                   | 12.0  | 2.45                                  | 0.39                                  |
| 10                  | 12.5  | 2.43                                  | 0.41                                  |



**Figure 6-17** Computer-generated (Minitab) control chart for Example 6-5.



**Figure 6-18** Standardized control chart for nonconformities per unit, Example 6-5.

As noted previously, the  $u$  chart should always be used when the sample size is variable. The most common implementation involves variable control limits, as illustrated in Example 6-5. There are, however, two other possible approaches:

1. Use control limits based on an average sample size

$$\bar{n} = \sum_{i=1}^m n_i / m$$

2. Use a standardized control chart (this is the preferred option).

This second alternative would involve plotting a standardized statistic

$$Z_i = \frac{u_i - \bar{u}}{\sqrt{\frac{\bar{u}}{n_i}}} \quad (6-20)$$

on a control chart with LCL =  $-3$  and UCL =  $+3$  and the center line at zero. This chart is appropriate if tests for runs and other pattern-recognition methods are to be used in conjunction with the chart. Figure 6-18 shows the standardized version of the control chart in Example 6-5. This standardized control chart could also be useful in the short production run situation (see Chapter 9, Section 9-1).

## 6-3 CONTROL CHARTS FOR NONCONFORMITIES (DEFECTS)

### 6-3.3 Demerit Systems

One possible demerit scheme is defined as follows.

**Class A Defects—Very Serious.** The unit is either completely unfit for service, or will fail in service in such a manner that cannot be easily corrected in the field, or will cause personal injury or property damage.

**Class B Defects—Serious.** The unit will possibly suffer a Class A operating failure, or will certainly cause somewhat less serious operating problems, or will certainly have reduced life or increased maintenance cost.

**Class C Defects—Moderately Serious.** The unit will possibly fail in service, or cause trouble that is less serious than operating failure, or possibly have reduced life or increased maintenance costs, or have a major defect in finish, appearance, or quality of work.

**Class D Defects—Minor.** The unit will not fail in service but has minor defects in finish, appearance, or quality of work.

Let  $c_{iA}$ ,  $c_{iB}$ ,  $c_{iC}$ , and  $c_{iD}$  represent the number of Class A, Class B, Class C, and Class D defects, respectively, in the  $i$ th inspection unit. We assume that each class of defect is independent, and the occurrence of defects in each class is well modeled by a Poisson distribution. Then we define the number of **demerits** in the inspection unit as

$$d_i = 100c_{iA} + 50c_{iB} + 10c_{iC} + c_{iD} \quad (6-21)$$

The demerit weights of Class A—100, Class B—50, Class C—10, and Class D—1 are used fairly widely in practice. However, any reasonable set of weights appropriate for a specific problem may also be used.

Suppose that a sample of  $n$  inspection units is used. Then the number of demerits per unit is

$$u_i = \frac{D}{n} \quad (6-22)$$

where  $D = \sum_{i=1}^n d_i$  is the total number of demerits in all  $n$  inspection units.

Since  $u_i$  is a linear combination of independent Poisson random variables, the statistics  $u_i$  could be plotted on a control chart with the following parameters:

$$\begin{aligned} \text{UCL} &= \bar{u} + 3\hat{\sigma}_u \\ \text{Center line} &= \bar{u} \\ \text{LCL} &= \bar{u} - 3\hat{\sigma}_u \end{aligned} \quad (6-23)$$

where

$$\bar{u} = 100\bar{u}_A + 50\bar{u}_B + 10\bar{u}_C + \bar{u}_D \quad (6-24)$$

and

$$\hat{\sigma}_u = \left[ \frac{(100)^2 \bar{u}_A + (50)^2 \bar{u}_B + (10)^2 \bar{u}_C + \bar{u}_D}{n} \right]^{1/2} \quad (6-25)$$

## 6-3 CONTROL CHARTS FOR NONCONFORMITIES (DEFECTS)

### 6-3.4 The Operating-Characteristic Function

The operating-characteristic (OC) curves for both the  $c$  chart and the  $u$  chart can be obtained from the Poisson distribution. For the  $c$  chart, the OC curve plots the probability of type II error  $\beta$  against the true mean number of defects  $c$ . The expression for  $\beta$  is

$$\beta = P\{x < \text{UCL}|c\} - P\{x \leq \text{LCL}|c\} \quad (6-26)$$

where  $x$  is a Poisson random variable with parameter  $c$ . Note that if the  $\text{LCL} < 0$  the second term on the right-hand side of equation 6-26 should be dropped.

We will generate the OC curve for the  $c$  chart in Example 6-3. For this example, since the LCL = 6.48 and the UCL = 33.22, equation 6-26 becomes

$$\beta = P\{x < 33.22|c\} - P\{x \leq 6.48|c\}$$

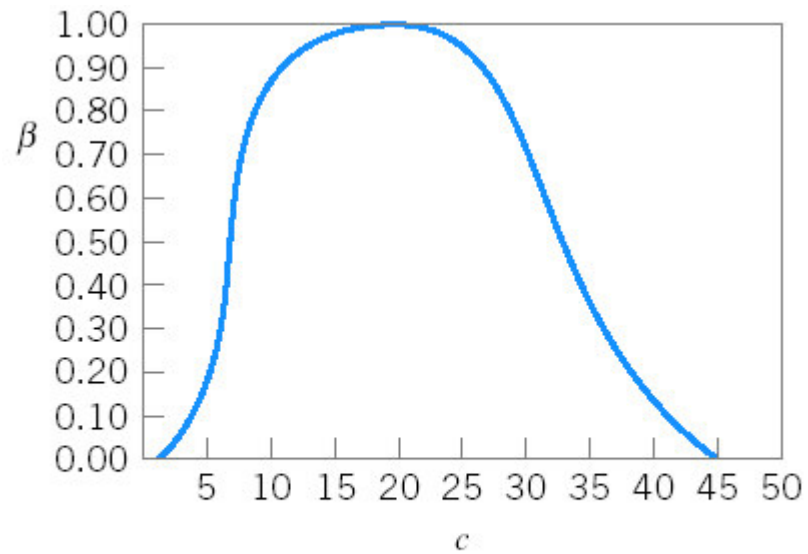
Since the number of nonconformities must be integer, this is equivalent to

$$\beta = P\{x \leq 33|c\} - P\{x \leq 6|c\}$$

These probabilities are evaluated in Table 6-13. The OC curve is shown in Fig. 6-19.

**Table 6-13** Calculation of the OC Curve for a  $c$  Chart with UCL = 33.22 and LCL = 6.48

| $c$ | $P\{x \leq 33 c\}$ | $P\{x \leq 6 c\}$ | $\beta = P\{x \leq 33 c\} - P\{x \leq 6 c\}$ |
|-----|--------------------|-------------------|----------------------------------------------|
| 1   | 1.000              | 0.999             | 0.001                                        |
| 3   | 1.000              | 0.966             | 0.034                                        |
| 5   | 1.000              | 0.762             | 0.238                                        |
| 7   | 1.000              | 0.450             | 0.550                                        |
| 10  | 1.000              | 0.130             | 0.870                                        |
| 15  | 0.999              | 0.008             | 0.991                                        |
| 20  | 0.997              | 0.000             | 0.997                                        |
| 25  | 0.950              | 0.000             | 0.950                                        |
| 30  | 0.744              | 0.000             | 0.744                                        |
| 33  | 0.546              | 0.000             | 0.546                                        |
| 35  | 0.410              | 0.000             | 0.410                                        |
| 40  | 0.151              | 0.000             | 0.151                                        |
| 45  | 0.038              | 0.000             | 0.038                                        |



**Figure 6-19** OC curve of a  $c$  chart with LCL = 6.48 and UCL = 33.22.

## 6-3 CONTROL CHARTS FOR NONCONFORMITIES (DEFECTS)

### 6-3.5 Dealing with Low Defect Levels

- When defect levels or in general, count rates, in a process become very low—say, under 1000 occurrences per million—there will be very long periods of time between the occurrence of a nonconforming unit. Conventional  $c$  and  $u$  charts become ineffective as count rates are driven into the low parts per million (ppm) range.
- The time-between-events control chart has been very effective as a process-control procedure for processes with low defect levels. If the “events” of interest occur according to a Poisson distribution, the probability distribution of the time between events is the exponential distribution.
- Constructing a time-between-events control chart is essentially equivalent to control charting an exponentially distributed variable.

Nelson (1994) has suggested solving this problem by transforming the exponential random variable to a Weibull random variable such that the resulting Weibull distribution is well approximated by the normal distribution. If  $y$  represents the original exponential random variable, the appropriate transformation is

$$x = y^{1/3.6} = y^{0.2777} \quad (6-28)$$

One would now construct a control chart on  $x$ , assuming that  $x$  follows a normal distribution.

..... **EXAMPLE 6-6** .....

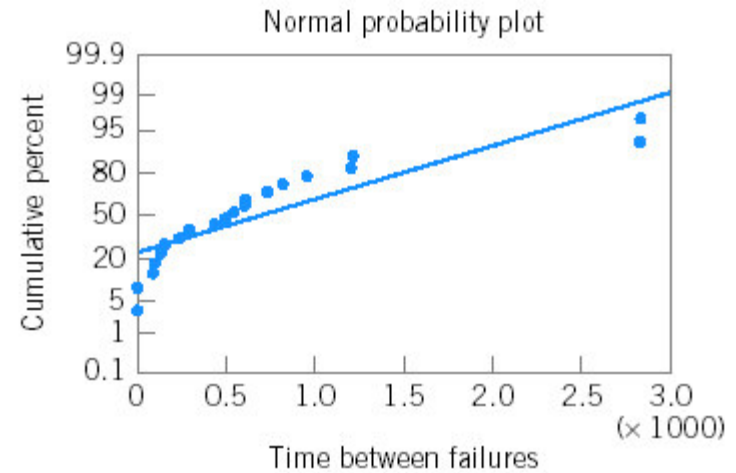
A chemical engineer wants to set up a control chart for monitoring the occurrence of failures of an important valve. She has decided to use the number of hours between failures as the variable to monitor. Table 6-14 shows the number of hours between failures for the last 20 failures of this valve. Figure 6-20 is a normal probability plot of the time between failures. Clearly, time between failures is not normally distributed.

Table 6-14 also shows the values of the transformed time between events, computed from equation 6-27. Figure 6-21 is a normal probability plot of the transformed time between failures. Note that the plot indicates that the distribution of this transformed variable is well approximated by the normal.

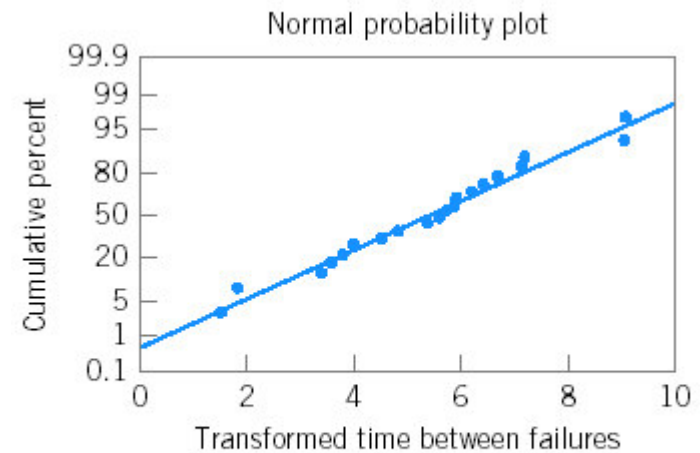
Figure 6-22 is a control chart for individuals and a moving range control chart for the transformed time between failures. Note that the control charts indicate a state of control, implying that the failure mechanism for this valve is constant. If a process change is made that improves the failure rate (such as a different type of maintenance action), then we would expect to see the mean time between failures get longer. This would result in points plotting above the upper control limit on the individuals control chart in Fig. 6-22.

**Table 6-14** Time between Failure Data, Example 6-6

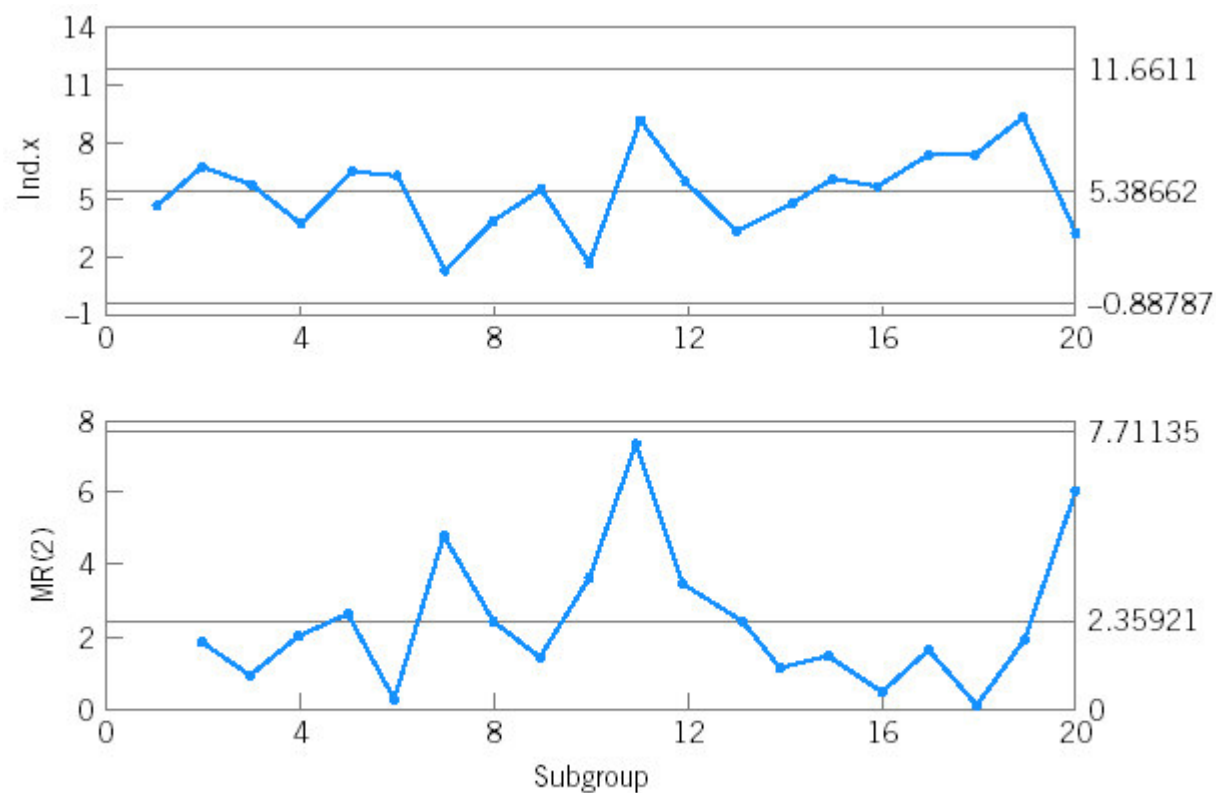
| Failure | Time between Failures, $y$ (hr) | Transformed Value of Time between Failures, $x = y^{0.2777}$ |
|---------|---------------------------------|--------------------------------------------------------------|
| 1       | 286                             | 4.80986                                                      |
| 2       | 948                             | 6.70903                                                      |
| 3       | 536                             | 5.72650                                                      |
| 4       | 124                             | 3.81367                                                      |
| 5       | 816                             | 6.43541                                                      |
| 6       | 729                             | 6.23705                                                      |
| 7       | 4                               | 1.46958                                                      |
| 8       | 143                             | 3.96768                                                      |
| 9       | 431                             | 5.39007                                                      |
| 10      | 8                               | 1.78151                                                      |
| 11      | 2837                            | 9.09619                                                      |
| 12      | 596                             | 5.89774                                                      |
| 13      | 81                              | 3.38833                                                      |
| 14      | 227                             | 4.51095                                                      |
| 15      | 603                             | 5.91690                                                      |
| 16      | 492                             | 5.59189                                                      |
| 17      | 1199                            | 7.16124                                                      |
| 18      | 1214                            | 7.18601                                                      |
| 19      | 2831                            | 9.09083                                                      |
| 20      | 96                              | 3.55203                                                      |



**Figure 6-20** Normal probability plot of time between failures, Example 6-6.



**Figure 6-21** Normal probability plot for the transformed failure data.



**Figure 6-22** Control charts for individuals and moving-range control chart for the transformed time between failures, Example 6-6.

## 6-5 GUIDELINES FOR IMPLEMENTING CONTROL CHARTS

Almost any process will benefit from SPC, including the use of control charts. In this section, we present some general guidelines helpful in implementing control charts. Specifically, we deal with the following:

1. Determining **which** process characteristics to control
2. Determining **where** the charts should be implemented in the process
3. Choosing the proper **type** of control charts
4. Taking **actions** to improve processes as the result of SPC/control chart analysis
5. Selecting data-collection systems and computer software

The guidelines are applicable to both variables and attributes control charts. Remember, control charts are not only for process surveillance; they should be used as an active, on-line method for reduction of process variability.

# Determining Which Characteristics to Control and Where to Put the Control Charts

At the start of a control chart program, it is usually difficult to determine which product or process characteristics should be controlled and at which points in the process to apply control charts. Some useful guidelines follow.

1. At the beginning of a control chart program, control charts should be applied to any product characteristics or manufacturing operations believed to be important. The charts will provide immediate feedback as to whether they are actually needed.
2. The control charts found to be unnecessary should be removed, and others that engineering and operator judgment indicates may be required should be added. More control charts will usually be employed at the beginning than after the process has stabilized.
3. Information on the number and types of control charts on the process should be kept current. It is best to keep separate records on the variables and attributes charts. In general, after the control charts are first installed, we often find that the number of control charts tends to increase rather steadily. After that, it will usually decrease. When the process stabilizes, we typically find that it has the same number of charts from one year to the next. However, they are not necessarily the same charts.

4. If control charts are being used effectively and if new knowledge is being gained about the key process variables, we should find that the number of  $\bar{x}$  and  $R$  charts increases and the number of attributes control charts decreases.
5. At the beginning of a control chart program there will usually be more attributes control charts, applied to semifinished or finished units near the *end* of the manufacturing process. As we learn more about the process, these charts will be replaced with  $\bar{x}$  and  $R$  charts applied *earlier* in the process to the critical parameters and operations that result in nonconformities in the finished product. Generally, **the earlier that process control can be established, the better**. In a complex assembly process, this may imply that process controls need to be implemented at the vendor or supplier level.
6. Control charts are an on-line, process-monitoring procedure. They should be implemented and maintained as close to the work center as possible, so that feedback will be rapid. Furthermore, the process operators and process engineering should have direct responsibility for collecting the process data, maintaining the charts, and interpreting the results. The operators and engineers have the detailed knowledge of the process required to correct process upsets and use the control chart to improve process performance. Microcomputers can speed up the feedback and should be an integral part of any modern, on-line, process-control procedure.
7. The out-of-control-action plan (OCAP) is a vital part of the control chart. Operating and engineering personnel should strive to keep OCAPs up-to-date and valid.

# Choosing the Proper Type of Control Chart

**A.  $\bar{x}$  and  $R$  (or  $\bar{x}$  and  $s$ ) charts.** Consider using variables control charts in these situations:

1. A new process is coming on stream, or a new product is being manufactured by an existing process.
2. The process has been in operation for some time, but it is chronically in trouble or unable to hold the specified tolerances.
3. The process is in trouble, and the control chart can be useful for diagnostic purposes (troubleshooting).
4. Destructive testing (or other expensive testing procedures) is required.
5. It is desirable to reduce acceptance-sampling or other downstream testing to a minimum when the process can be operated in control.
6. Attributes control charts have been used, but the process is either out of control or in control but the yield is unacceptable.
7. There are very tight specifications, overlapping assembly tolerances, or other difficult manufacturing problems.
8. The operator must decide whether or not to adjust the process, or when a setup must be evaluated.
9. A change in product specifications is desired.
10. Process stability and capability must be continually demonstrated, such as in regulated industries.

**B. Attributes Charts ( $p$  charts,  $c$  charts, and  $u$  charts).** Consider using attributes control charts in these situations:

1. Operators control the assignable causes, and it is necessary to reduce process fall-out.
2. The process is a complex assembly operation and product quality is measured in terms of the occurrence of nonconformities, successful or unsuccessful product function, and so forth. (Examples include computers, office automation equipment, automobiles, and the major subsystems of these products.)
3. Process control is necessary, but measurement data cannot be obtained.
4. A historical summary of process performance is necessary. Attributes control charts, such as  $p$  charts,  $c$  charts, and  $u$  charts, are very effective for summarizing information about the process for management review.
5. Remember that attributes charts are generally inferior to charts for variables. Always use  $\bar{x}$  and  $R$  or  $\bar{x}$  and  $s$  charts whenever possible.

**C. Control Charts for Individuals.** Consider using the control chart for individuals in conjunction with a moving-range chart in these situations:

1. It is inconvenient or impossible to obtain more than one measurement per sample, or repeat measurements will only differ by laboratory or analysis error. Examples often occur in chemical processes.
2. Automated testing and inspection technology allow measurement of every unit produced. In these cases, also consider the cumulative sum control chart and the exponentially weighted moving average control chart discussed in Chapter 7.
3. The data become available very slowly, and waiting for a larger sample will be impractical or make the control procedure too slow to react to problems. This often happens in nonproduct situations; for example, accounting data may become available only monthly.
4. Generally, once we are in phase II, individuals charts have poor performance in shift detection and can be very sensitive to departures from normality. Always use the EWMA and cusum charts of Chapter 8 in phase II instead of individuals charts whenever possible.

# Actions Taken to Improve the Process

Figure 6-27 gives the answers to two questions: “Is the process in control?” and “Is the process capable (in the sense of the previous paragraph)?” Each of the four cells in the figure contains some recommended courses of action that depend on the answers to these two questions.

|                                        |     | IS THE PROCESS CAPABLE? |                                                                            |
|----------------------------------------|-----|-------------------------|----------------------------------------------------------------------------|
|                                        |     | Yes                     | No                                                                         |
| IS<br>THE<br>PROCESS<br>IN<br>CONTROL? | Yes | SPC                     | SPC<br>Experimental design<br>Investigate specifications<br>Change process |
|                                        | No  | SPC                     | SPC<br>Experimental design<br>Investigate specifications<br>Change process |

**Figure 6-27** Actions taken to improve a process.

The lower two boxes in Fig. 6-27 deal with the case of an out-of-control process. The southeast corner presents the case of a process that is out of control and not capable. (Remember our nontechnical use of the term *capability*.) The actions recommended here are identical to those for the box in the northeast corner, except that SPC would be expected to yield fairly rapid results now, because the control charts should be identifying the presence of assignable causes. The other methods of attack will warrant consideration and use in many cases, however. Finally, the southwest corner treats the case of a process that exhibits lack of statistical control but does not produce a meaningful number of defectives because the specifications are very wide. SPC methods should still be used to establish control and reduce variability in this case, for the following reasons:

1. Specifications can change without notice.
2. The customer may require both **control** and **capability**.
3. The fact that the process experiences assignable causes implies that unknown forces are at work; these unknown forces could result in poor capability in the near future.

## IMPORTANT TERMS AND CONCEPTS

---

|                                                                       |                                                              |
|-----------------------------------------------------------------------|--------------------------------------------------------------|
| Attribute data                                                        | Demerit systems for attribute data                           |
| Average run length for attribute control charts                       | Design of attributes control charts                          |
| Cause-and-effect diagram                                              | Fraction defective                                           |
| Choice between attributes and variables data                          | Fraction nonconforming                                       |
| Control chart for defects or nonconformities<br>per unit or $u$ chart | Nonconformities                                              |
| Control chart for fraction nonconforming<br>or $p$ chart              | Operating characteristic curve<br>for the $c$ and $u$ charts |
| Control chart for nonconformities or $c$ chart                        | Operating characteristic curve for the $p$ chart             |
| Control chart for number nonconforming<br>or $np$ chart               | Pareto chart                                                 |
| Defect                                                                | Standardized control charts                                  |
| Defective                                                             | Time between occurrence control charts                       |
|                                                                       | Variable sample size for attributes control chart            |

# LEARNING OBJECTIVES

1. Understand the statistical basis of attributes control charts
2. Know how to design attributes control charts
3. Know how to set up and use the  $p$  chart for fraction nonconforming
4. Know how to set up and use the  $np$  control chart for the number of nonconforming items
5. Know how to set up and use the  $c$  control chart for defects
6. Know how to set up and use the  $u$  control chart for defects per unit
7. Use attributes control charts with variable sample size
8. Understand the advantages and disadvantages of attributes versus variables control charts
9. Understand the rational subgroup concept for attributes control charts
10. Determine the average run length for attributes control charts