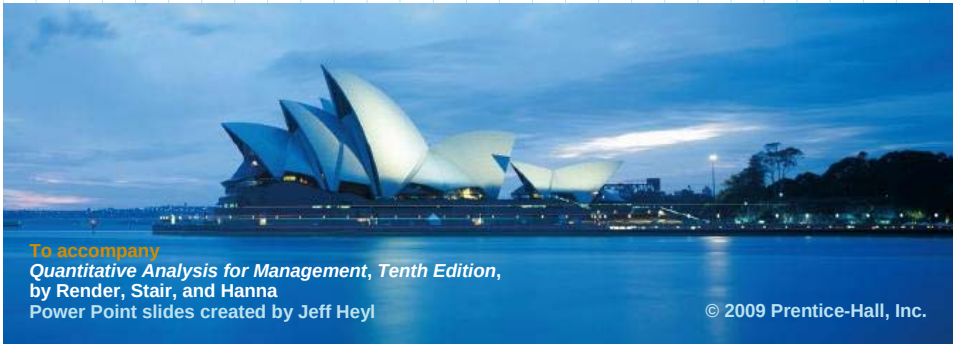


Chapter 8

LP Modeling Applications with Computer Analyses in Excel and QM for Windows



Chapter Outline

- 8.1 Introduction
- 8.2 Marketing Applications
- 8.3 Manufacturing Applications
- 8.4 Employee Scheduling Applications
- 8.5 Financial Applications
- 8.6 Transportation Applications
- 8.7 Transshipment Applications
- 8.8 Ingredient Blending Applications

Learning Objectives

After completing this chapter, students will be able to:

1. Model a wide variety of medium to large LP problems
2. Understand major application areas, including marketing, production, labor scheduling, fuel blending, transportation, and finance
3. Gain experience in solving LP problems with QM for Windows and Excel Solver software

Introduction

- The graphical method of LP is useful for understanding how to formulate and solve small LP problems
- There are many types of problems that can be solved using LP
- The principles developed here are applicable to larger problems

Marketing Applications

- Linear programming models have been used in the advertising field as a decision aid in selecting an effective media mix
- Media selection problems can be approached with LP from two perspectives
 - Maximize audience exposure
 - Minimize advertising costs

Marketing Applications

- Win Big Gambling Club advertising options

MEDIUM	AUDIENCE REACHED PER AD	COST PER AD (\$)	MAXIMUM ADS PER WEEK
TV spot (1 minute)	5,000	800	12
Daily newspaper (full-page ad)	8,500	925	5
Radio spot (30 seconds, prime time)	2,400	290	25
Radio spot (1 minute, afternoon)	2,800	380	20

Marketing Applications

- The Win Big Gambling Club promotes gambling junkets to the Bahamas
- They have \$8,000 per week to spend on advertising
- Their goal is to reach the largest possible high-potential audience
- Media types and audience figures are shown in the following table
- They need to place at least five radio spots per week
- No more than \$1,800 can be spent on radio advertising each week

Win Big Gambling Club

- The problem formulation is

X_1 = number of 1-minute TV spots each week

X_2 = number of daily paper ads each week

X_3 = number of 30-second radio spots each week

X_4 = number of 1-minute radio spots each week

Objective:

Maximize audience coverage

$$= 5,000X_1 + 8,500X_2 + 2,400X_3 + 2,800X_4$$

Subject to

$$X_1 \leq 12 \quad (\text{max TV spots/wk})$$

$$X_2 \leq 5 \quad (\text{max newspaper ads/wk})$$

$$X_3 \leq 25 \quad (\text{max 30-sec radio spots ads/wk})$$

$$X_4 \leq 20 \quad (\text{max newspaper ads/wk})$$

$$800X_1 + 925X_2 + 290X_3 + 380X_4 \leq \$8,000 \quad (\text{weekly advertising budget})$$

$$X_3 + X_4 \geq 5 \quad (\text{min radio spots contracted})$$

$$290X_3 + 380X_4 \leq \$1,800 \quad (\text{max dollars spent on radio})$$

$$X_1, X_2, X_3, X_4 \geq 0$$

Win Big Gambling Club

	A	B	C	D	E	F	G	H
1	Win Big Gambling Club							
2		1 minute	newspaper	radio	1 minute			
3		TV spots	ads	spots	radio spots			
4	Solution	1	1	1	1			
5	Variables	X1	X2	X3	X4			RHS
6	Audience reached per ad	5000	8500	2400	2800	=SUMPRODUCT(\$B\$4:\$E\$4,B6:E6)		
7	Maximum TV	1				=SUMPRODUCT(\$B\$4:\$E\$4,B7:E7)	<=	12
8	Maximum Newspaper		1			=SUMPRODUCT(\$B\$4:\$E\$4,B8:E8)	<=	5
9	Maximum 30-second radio			1		=SUMPRODUCT(\$B\$4:\$E\$4,B9:E9)	<=	25
10	Maximum 1 min. radio				1	=SUMPRODUCT(\$B\$4:\$E\$4,B10:E10)	<=	20
11	Cost per ad	800	925	290	380	=SUMPRODUCT(\$B\$4:\$E\$4,B11:E11)	<=	8000
12	Radio dollars			290	380	=SUMPRODUCT(\$B\$4:\$E\$4,B12:E12)	<=	1800
13	Radio spots			1	1	=SUMPRODUCT(\$B\$4:\$E\$4,B13:E13)	>=	5

Solver Parameters

Set Target Cell:

Equal To: ☒ Max ☐ Min ☐ Value of:

By Changing Variable Cells:

Subject to the Constraints:

Program 8.1A

Marketing Research

- Linear programming has also been applied to marketing research problems and the area of consumer research
- Statistical pollsters can use LP to help make strategy decisions

Win Big Gambling Club

■ The problem solution

	A	B	C	D	E	F	G	H
1	Win Big Gambling Club							
2		1 minute	newspaper	30 second	1 minute			
3		TV spots	ads	radio spots	radio spots			
4	Solution	1.96875	5	6.2068966	0			
5	Variables	X1	X2	X3	X4			RHS
6	Audience reached per ad	5000	8500	2400	2800	67240.3		
7	Maximum TV	1				1.96875	<=	12
8	Maximum Newspaper		1			5	<=	5
9	Maximum 30-second radio			1		6.206897	<=	25
10	Maximum 1 min. radio				1	0	<=	20
11	Cost per ad	800	925	290	380	8000	<=	8000
12	Radio dollars			290	380	1800	<=	1800
13	Radio spots			1	1	6.206897	>=	5

Program 8.1B

Marketing Research

- Management Sciences Associates (MSA) is a marketing research firm
- MSA determines that it must fulfill several requirements in order to draw statistically valid conclusions
 - Survey at least 2,300 U.S. households
 - Survey at least 1,000 households whose heads are 30 years of age or younger
 - Survey at least 600 households whose heads are between 31 and 50 years of age
 - Ensure that at least 15% of those surveyed live in a state that borders on Mexico
 - Ensure that no more than 20% of those surveyed who are 51 years of age or over live in a state that borders on Mexico

Marketing Research

- MSA decides that all surveys should be conducted in person
- It estimates the costs of reaching people in each age and region category are as follows

REGION	COST PER PERSON SURVEYED (\$)		
	AGE ≤ 30	AGE 31-50	AGE ≥ 51
State bordering Mexico	\$7.50	\$6.80	\$5.50
State not bordering Mexico	\$6.90	\$7.25	\$6.10

Marketing Research

Objective function

$$\begin{aligned} \text{Minimize total interview costs} &= \$7.50X_1 + \$6.80X_2 + \$5.50X_3 \\ &\quad + \$6.90X_4 + \$7.25X_5 + \$6.10X_6 \end{aligned}$$

subject to

$$\begin{aligned} X_1 + X_2 + X_3 + X_4 + X_5 + X_6 &\geq 2,300 \quad (\text{total households}) \\ X_1 + X_4 &\geq 1,000 \quad (\text{households 30 or younger}) \\ X_2 + X_5 &\geq 600 \quad (\text{households 31-50}) \\ X_1 + X_2 + X_3 &\geq 0.15(X_1 + X_2 + X_3 + X_4 + X_5 + X_6) \quad (\text{border states}) \\ X_3 &\leq 0.20(X_3 + X_6) \quad (\text{limit on age group 51+ who can live in border state}) \\ X_1, X_2, X_3, X_4, X_5, X_6 &\geq 0 \end{aligned}$$

Marketing Research

- MSA's goal is to meet the sampling requirements at the least possible cost
- The decision variables are

X_1 = number of 30 or younger and in a border state
 X_2 = number of 31-50 and in a border state
 X_3 = number 51 or older and in a border state
 X_4 = number 30 or younger and not in a border state
 X_5 = number of 31-50 and not in a border state
 X_6 = number 51 or older and not in a border state

Marketing Research

- Computer solution in QM for Windows
- Notice the variables in the constraints have all been moved to the left side of the equations

	X1	X2	X3	X4	X5	X6	RHS	Dual
Minimize	7.5	6.8	5.5	6.9	7.25	6.1		
Constraint 1	1.	1.	1.	1.	1.	1.	>= 2,300.	-5.98
Constraint 2	1.	0.	0.	1.	0.	0.	>= 1,000.	-0.92
Constraint 3	0.	1.	0.	0.	1.	0.	>= 600.	-0.82
Constraint 4	0.85	0.85	0.85	-0.15	-0.15	-0.15	>= 0.	0.
Constraint 5	0.	0.	0.8	0.	0.	-0.2	<= 0.	0.6
Solution->	0.	600.	140.	1,000.	0.	559.9999	15,166.	

Program 8.2

Marketing Research

- The following table summarizes the results of the MSA analysis
- It will cost MSA \$15,166 to conduct this research

REGION	AGE ≤ 30	AGE 31-50	AGE ≥ 51
State bordering Mexico	0	600	140
State not bordering Mexico	1,000	0	560

Manufacturing Applications

- Fifth Avenue Industries produces four varieties of ties
 - One is expensive all-silk
 - One is all-polyester
 - Two are polyester and cotton blends
- The table on the below shows the cost and availability of the three materials used in the production process

MATERIAL	COST PER YARD (\$)	MATERIAL AVAILABLE PER MONTH (YARDS)
Silk	21	800
Polyester	6	3,000
Cotton	9	1,600

Manufacturing Applications

- Production Mix
 - LP can be used to plan the optimal mix of products to manufacture
 - Company must meet a myriad of constraints, ranging from financial concerns to sales demand to material contracts to union labor demands
 - Its primary goal is to generate the largest profit possible

Manufacturing Applications

- The firm has contracts with several major department store chains to supply ties
- Contracts require a minimum number of ties but may be increased if demand increases
- Fifth Avenue's goal is to maximize monthly profit given the following decision variables

X_1 = number of all-silk ties produced per month
 X_2 = number polyester ties
 X_3 = number of blend 1 poly-cotton ties
 X_4 = number of blend 2 poly-cotton ties

Manufacturing Applications

Contract data for Fifth Avenue Industries

VARIETY OF TIE	SELLING PRICE PER TIE (\$)	MONTHLY CONTRACT MINIMUM	MONTHLY DEMAND	MATERIAL REQUIRED PER TIE (YARDS)	MATERIAL REQUIREMENTS
All silk	6.70	6,000	7,000	0.125	100% silk
All polyester	3.55	10,000	14,000	0.08	100% polyester
Poly-cotton blend 1	4.31	16,000	16,000	0.10	50% polyester-50% cotton
Poly-cotton blend 2	4.81	8,500	8,500	0.10	30% polyester-70% cotton

Table 8.1

Manufacturing Applications

The complete Fifth Avenue Industries model

Objective function

Maximize profit = $\$4.08X_1 + \$3.07X_2 + \$3.56X_3 + \$4.00X_4$

Subject to

$0.125X_1 \leq 800$ (yds of silk)

$0.08X_2 + 0.05X_3 + 0.03X_4 \leq 3,000$ (yds of polyester)

$0.05X_3 + 0.07X_4 \leq 1,600$ (yds of cotton)

$X_1 \geq 6,000$ (contract min for silk)

$X_1 \leq 7,000$ (contract min)

$X_2 \geq 10,000$ (contract min for all polyester)

$X_2 \leq 14,000$ (contract max)

$X_3 \geq 13,000$ (contract mini for blend 1)

$X_3 \leq 16,000$ (contract max)

$X_4 \geq 6,000$ (contract mini for blend 2)

$X_4 \leq 8,500$ (contract max)

$X_1, X_2, X_3, X_4 \geq 0$

Manufacturing Applications

Fifth Avenue also has to calculate profit per tie for the objective function

VARIETY OF TIE	SELLING PRICE PER TIE (\$)	MATERIAL REQUIRED PER TIE (YARDS)	MATERIAL COST PER YARD (\$)	COST PER TIE (\$)	PROFIT PER TIE (\$)
All silk	\$6.70	0.125	\$21	\$2.62	\$4.08
All polyester	\$3.55	0.08	\$6	\$0.48	\$3.07
Poly-cotton blend 1	\$4.31	0.05	\$6	\$0.30	
		0.05	\$9	\$0.45	\$3.56
Poly-cotton blend 2	\$4.81	0.03	\$6	\$0.18	
		0.70	\$9	\$0.63	\$4.00

Manufacturing Applications

Excel formulation for Fifth Avenue LP problem

The variables are in a column rather than a row for this example.

This cell contains the formula for total profit.

Calculate total material used.

This is the material data.

Do not make more than what is demanded (4 constraints).

Meet the monthly minimums (4 constraints).

Do not use more material than is available (3 constraints).

Program 8.3A

Manufacturing Applications

■ Solution for Fifth Avenue Industries LP model

	A	B	C	D	E	F	G	H	I
1	Fifth Avenue Industries								
2									
3	Variety	Number (X)	Selling price	Monthly minimum	Monthly demand	Material (yards)	silk	polyester	cotton
4	All silk	6400	6.7	6000	7000	0.125	100%		
5	All polyester	14000	3.55	10000	14000	0.08		100%	
6	Poly-cotton blend 1	16000	4.31	13000	16000	0.1		50%	50%
7	Poly-cotton blend 2	9500	4.81	6000	8500	0.1		30%	70%
8									
9		Total revenue	202425				800	2175	1395
10									
11	Material	Cost	Available	Used					
12	Silk	21	800	800					
13	Polyester	6	3000	2175					
14	Cotton	9	1600	1395					
15									
16	Total Cost			42405					
17									
18	Total Profit			160020					

Program 8.3B

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Manufacturing Applications

- Greenberg Motors, Inc. manufactures two different electric motors for sale under contract to Drexel Corp.
- Drexel places orders three times a year for four months at a time
- Demand varies month to month as shown below
- Greenberg wants to develop its production plan for the next four months

MODEL	JANUARY	FEBRUARY	MARCH	APRIL
GM3A	800	700	1,000	1,100
GM3B	1,000	1,200	1,400	1,400

Table 8.2

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Manufacturing Applications

■ Production Scheduling

- Setting a low-cost production schedule over a period of weeks or months is a difficult and important management task
- Important factors include labor capacity, inventory and storage costs, space limitations, product demand, and labor relations
- When more than one product is produced, the scheduling process can be quite complex
- The problem resembles the product mix model for each time period in the future

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Manufacturing Applications

- Production planning at Greenberg must consider four factors
 - Desirability of producing the same number of motors each month to simplify planning and scheduling
 - Necessity to inventory carrying costs down
 - Warehouse limitations
 - The no-lay-off policy
- LP is a useful tool for creating a minimum total cost schedule the resolves conflicts between these factors

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Manufacturing Applications

- Double subscripted variables are used in this problem to denote motor type and month of production

$X_{A,i}$ = Number of model GM3A motors produced in month i ($i = 1, 2, 3, 4$ for January – April)

$X_{B,i}$ = Number of model GM3B motors produced in month i

- It costs \$10 to produce a GM3A motor and \$6 to produce a GM3B
- Both costs increase by 10% on March 1, thus

$$\text{Cost of production} = \$10X_{A1} + \$10X_{A2} + \$11X_{A3} + \$11X_{A4} \\ + \$6X_{B1} + \$6X_{B2} + \$6.60X_{B3} + \$6.60X_{B4}$$

Manufacturing Applications

- We combine these two for the objective function

$$\begin{aligned} \text{Minimize total cost} = & \$10X_{A1} + \$10X_{A2} + \$11X_{A3} + \$11X_{A4} \\ & + \$6X_{B1} + \$6X_{B2} + \$6.60X_{B3} + \$6.60X_{B4} \\ & + \$0.18X_{A1} + \$0.18X_{A2} + \$0.18X_{A3} + \$0.18X_{A4} \\ & + \$0.13X_{B1} + \$0.13X_{B2} + \$0.13X_{B3} + \$0.13X_{B4} \end{aligned}$$

- End of month inventory is calculated using this relationship

$$\left(\begin{array}{c} \text{Inventory} \\ \text{at the end} \\ \text{of this} \\ \text{month} \end{array} \right) = \left(\begin{array}{c} \text{Inventory} \\ \text{at the end} \\ \text{of last} \\ \text{month} \end{array} \right) + \left(\begin{array}{c} \text{Current} \\ \text{month's} \\ \text{production} \end{array} \right) - \left(\begin{array}{c} \text{Sales to} \\ \text{Drexel this} \\ \text{month} \end{array} \right)$$

Manufacturing Applications

- We can use the same approach to create the portion of the objective function dealing with inventory carrying costs

$I_{A,i}$ = Level of on-hand inventory for GM3A motors at the end of month i ($i = 1, 2, 3, 4$ for January – April)

$I_{B,i}$ = Level of on-hand inventory for GM3B motors at the end of month i

- The carrying cost for GM3A motors is \$0.18 per month and the GM3B costs \$0.13 per month
- Monthly ending inventory levels are used for the average inventory level

$$\begin{aligned} \text{Cost of carrying inventory} = & \$0.18X_{A1} + \$0.18X_{A2} + \$0.18X_{A3} + \$0.18X_{A4} \\ & + \$0.13X_{B1} + \$0.13X_{B2} + \$0.13X_{B3} + \$0.13X_{B4} \end{aligned}$$

Manufacturing Applications

- Greenberg is starting a new four-month production cycle with a change in design specification that left no old motors in stock on January 1
- Given January demand for both motors

$$\begin{aligned} I_{A1} &= 0 + X_{A1} - 800 \\ I_{B1} &= 0 + X_{B1} - 1,000 \end{aligned}$$

- Rewritten as January's constraints

$$\begin{aligned} X_{A1} - I_{A1} &= 800 \\ X_{B1} - I_{B1} &= 1,000 \end{aligned}$$

Manufacturing Applications

■ Constraints for February, March, and April

$$\begin{array}{ll} X_{A2} + I_{A1} - I_{A2} = 700 & \text{February GM3A demand} \\ X_{B2} + I_{B1} - I_{B2} = 1,200 & \text{February GM3B demand} \\ X_{A3} + I_{A2} - I_{A3} = 1,000 & \text{March GM3A demand} \\ X_{B3} + I_{B2} - I_{B3} = 1,400 & \text{March GM3B demand} \\ X_{A4} + I_{A3} - I_{A4} = 1,100 & \text{April GM3A demand} \\ X_{B4} + I_{B3} - I_{B4} = 1,400 & \text{April GM3B demand} \end{array}$$

■ And constraints for April's ending inventory

$$\begin{array}{l} I_{A4} = 450 \\ I_{B4} = 300 \end{array}$$

Manufacturing Applications

■ Labor hour constraints

$$\begin{array}{ll} 1.3X_{A1} + 0.9X_{B1} \geq 2,240 & \text{(January min hrs/month)} \\ 1.3X_{A1} + 0.9X_{B1} \leq 2,560 & \text{(January max hrs/month)} \\ 1.3X_{A2} + 0.9X_{B2} \geq 2,240 & \text{(February labor min)} \\ 1.3X_{A2} + 0.9X_{B2} \leq 2,560 & \text{(February labor max)} \\ 1.3X_{A3} + 0.9X_{B3} \geq 2,240 & \text{(March labor min)} \\ 1.3X_{A3} + 0.9X_{B3} \leq 2,560 & \text{(March labor max)} \\ 1.3X_{A4} + 0.9X_{B4} \geq 2,240 & \text{(April labor min)} \\ 1.3X_{A4} + 0.9X_{B4} \leq 2,560 & \text{(April labor max)} \\ \text{All variables} \geq 0 & \text{Nonnegativity constraints} \end{array}$$

Manufacturing Applications

■ We also need constraints for warehouse space

$$\begin{array}{l} I_{A1} + I_{B1} \leq 3,300 \\ I_{A2} + I_{B2} \leq 3,300 \\ I_{A3} + I_{B3} \leq 3,300 \\ I_{A4} + I_{B4} \leq 3,300 \end{array}$$

- No worker is ever laid off so Greenberg has a base employment level of 2,240 labor hours per month
- By adding temporary workers, available labor hours can be increased to 2,560 hours per month
- Each GM3A motor requires 1.3 labor hours and each GM3B requires 0.9 hours

Manufacturing Applications

■ Greenberg Motors solution

PRODUCTION SCHEDULE	JANUARY	FEBRUARY	MARCH	APRIL
Units GM3A produced	1,277	1,138	842	792
Units GM3B produced	1,000	1,200	1,400	1,700
Inventory GM3A carried	477	915	758	450
Inventory GM3B carried	0	0	0	300
Labor hours required	2,560	2,560	2,355	2,560

- Total cost for this four month period is \$76,301.61
- Complete model has 16 variables and 22 constraints

Employee Scheduling Applications

■ Assignment Problems

- Involve determining the most efficient way to assign resources to tasks
- Objective may be to minimize travel times or maximize assignment effectiveness
- Assignment problems are unique because they have a coefficient of 0 or 1 associated with each variable in the LP constraints and the right-hand side of each constraint is always equal to 1

Employee Scheduling Applications

■ Effectiveness ratings

LAWYER	CLIENT'S CASE			
	DIVORCE	CORPORATE MERGER	EMBEZZLEMENT	EXHIBITIONISM
Adams	6	2	8	5
Brooks	9	3	5	8
Carter	4	8	3	4
Darwin	6	7	6	4

Let $X_{ij} = \begin{cases} 1 & \text{if attorney } i \text{ is assigned to case } j \\ 0 & \text{otherwise} \end{cases}$

where $i = 1, 2, 3, 4$ stands for Adams, Brooks, Carter, and Darwin respectively
 $j = 1, 2, 3, 4$ stands for divorce, merger, embezzlement, and exhibitionism

Employee Scheduling Applications

- Ivan and Ivan law firm maintains a large staff of young attorneys
- Ivan wants to make lawyer-to-client assignments in the most effective manner
- He identifies four lawyers who could possibly be assigned new cases
- Each lawyer can handle one new client
- The lawyers have different skills and special interests
- The following table summarizes the lawyers estimated effectiveness on new cases

Employee Scheduling Applications

■ The LP formulation is

$$\begin{aligned} \text{Maximize effectiveness} = & 6X_{11} + 2X_{12} + 8X_{13} + 5X_{14} + 9X_{21} + 3X_{22} \\ & + 5X_{23} + 8X_{24} + 4X_{31} + 8X_{32} + 3X_{33} + 4X_{34} \\ & + 6X_{41} + 7X_{42} + 6X_{43} + 4X_{44} \end{aligned}$$

$$\begin{aligned} \text{subject to } & X_{11} + X_{21} + X_{31} + X_{41} = 1 && \text{(divorce case)} \\ & X_{12} + X_{22} + X_{32} + X_{42} = 1 && \text{(merger)} \\ & X_{13} + X_{23} + X_{33} + X_{43} = 1 && \text{(embezzlement)} \\ & X_{14} + X_{24} + X_{34} + X_{44} = 1 && \text{(exhibitionism)} \\ & X_{11} + X_{12} + X_{13} + X_{14} = 1 && \text{(Adams)} \\ & X_{21} + X_{22} + X_{23} + X_{24} = 1 && \text{(Brook)} \\ & X_{31} + X_{32} + X_{33} + X_{34} = 1 && \text{(Carter)} \\ & X_{41} + X_{42} + X_{43} + X_{44} = 1 && \text{(Darwin)} \end{aligned}$$

Employee Scheduling Applications

- Solving Ivan and Ivan's assignment scheduling LP problem using QM for Windows

QM for Windows - C:\Prentice\Data\ReaderStar7\Ivan.LIN

Objective
☒ Maximize
☐ Minimize

Linear Programming Results

Ivan and Ivan Solution

	x11	x12	x13	x14	x21	x22	x23	x24	x31	x32	x33	x34	x41	x42	x43	x44	RHS	Dual
Maximize	6.	2.	8.	5.	9.	3.	5.	8.	4.	8.	3.	4.	6.	7.	6.	4.		
Divorce	1.	1.	1.	1.													= 1.	5.
Merger					1.	1.	1.	1.									= 1.	8.
Embezzlement									1.	1.	1.	1.					= 1.	4.
Exhibitionism													1.	1.	1.	1.	= 1.	5.
Adams	1.				1.				1.				1.				= 1.	1.
Brooks		1.				1.				1.				1.			= 1.	4.
Carter			1.				1.				1.				1.		= 1.	3.
Darwin				1.				1.				1.				1.	= 1.	0.
Solution->	0.	0.	1.	0.	0.	0.	0.	1.	0.	1.	0.	0.	1.	0.	0.	0.	30.	

Program 8.4

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Employee Scheduling Applications

- Hong Kong Bank of Commerce and Industry has requirements for between 10 and 18 tellers depending on the time of day
- Lunch time from noon to 2 pm is generally the busiest
- The bank employs 12 full-time tellers but has many part-time workers available
- Part-time workers must put in exactly four hours per day, can start anytime between 9 am and 1 pm, and are inexpensive
- Full-time workers work from 9 am to 3 pm and have 1 hour for lunch

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Employee Scheduling Applications

- Labor Planning
 - Addresses staffing needs over a particular time
 - Especially useful when there is some flexibility in assigning workers that require overlapping or interchangeable talents

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Employee Scheduling Applications

- Labor requirements for Hong Kong Bank of Commerce and Industry

TIME PERIOD	NUMBER OF TELLERS REQUIRED
9 am - 10 am	10
10 am - 11 am	12
11 am - Noon	14
Noon - 1 pm	16
1 pm - 2 pm	18
2 pm - 3 pm	17
3 pm - 4 pm	15
4 pm - 5 pm	10

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Employee Scheduling Applications

- Part-time hours are limited to a maximum of 50% of the day's total requirements
- Part-timers earn \$8 per hour on average
- Full-timers earn \$100 per day on average
- The bank wants a schedule that will minimize total personnel costs
- It will release one or more of its part-time tellers if it is profitable to do so

Employee Scheduling Applications

■ Objective function

Minimize total daily personnel cost = $\$100F + \$32(P_1 + P_2 + P_3 + P_4 + P_5)$

subject to

$$\begin{array}{rcll}
 F + P_1 & & \geq 10 & \text{(9 am - 10 am needs)} \\
 F + P_1 + P_2 & & \geq 12 & \text{(10 am - 11 am needs)} \\
 0.5F + P_1 + P_2 + P_3 & & \geq 14 & \text{(11 am - noon needs)} \\
 0.5F + P_1 + P_2 + P_3 + P_4 & & \geq 16 & \text{(noon - 1 pm needs)} \\
 F + P_2 + P_3 + P_4 + P_5 & \geq 18 & & \text{(1 pm - 2 pm needs)} \\
 F + P_3 + P_4 + P_5 & \geq 17 & & \text{(2 pm - 3 pm needs)} \\
 F + P_4 + P_5 & \geq 15 & & \text{(3 pm - 4 pm needs)} \\
 F + P_5 & \geq 10 & & \text{(4 pm - 5 pm needs)} \\
 F & \leq 12 & & \text{(12 full-time tellers)} \\
 4P_1 + 4P_2 + 4P_3 + 4P_4 + 4P_5 & \leq 0.50(112) & & \text{(max 50\% part-timers)} \\
 P_1, P_2, P_3, P_4, P_5 & \geq 0 & &
 \end{array}$$

Employee Scheduling Applications

■ We let

F = full-time tellers

P_1 = part-timers starting at 9 am (leaving at 1 pm)

P_2 = part-timers starting at 10 am (leaving at 2 pm)

P_3 = part-timers starting at 11 am (leaving at 3 pm)

P_4 = part-timers starting at noon (leaving at 4 pm)

P_5 = part-timers starting at 1 pm (leaving at 5 pm)

Employee Scheduling Applications

■ There are several alternate optimal schedules Hong Kong Bank can follow

■ $F = 10, P_2 = 2, P_3 = 7, P_4 = 5, P_1, P_5 = 0$

■ $F = 10, P_1 = 6, P_2 = 1, P_3 = 2, P_4 = 5, P_5 = 0$

■ The cost of either of these two policies is \$1,448 per day

Financial Applications

■ Portfolio Selection

- Bank, investment funds, and insurance companies often have to select specific investments from a variety of alternatives
- The manager's overall objective is generally to maximize the potential return on the investment given a set of legal, policy, or risk restraints

Financial Applications

- ICT has \$5 million to invest and wants to accomplish two things
- Maximize the return on investment over the next six months
- Satisfy the diversification requirements set by the board
- The board has also decided that at least 55% of the funds must be invested in gold stocks and construction loans and no less than 15% be invested in trade credit

Financial Applications

- International City Trust (ICT) invests in short-term trade credits, corporate bonds, gold stocks, and construction loans
- The board of directors has placed limits on how much can be invested in each area

INVESTMENT	INTEREST EARNED (%)	MAXIMUM INVESTMENT (\$ MILLIONS)
Trade credit	7	1.0
Corporate bonds	11	2.5
Gold stocks	19	1.5
Construction loans	15	1.8

Financial Applications

- The variables in the model are

X_1 = dollars invested in trade credit

X_2 = dollars invested in corporate bonds

X_3 = dollars invested in gold stocks

X_4 = dollars invested in construction loans

Financial Applications

■ Objective function

Maximize
dollars of
interest
earned

$$= 0.07X_1 + 0.11X_2 + 0.19X_3 + 0.15X_4$$

subject to

$$\begin{array}{rcl} X_1 & \leq & 1,000,000 \\ X_2 & \leq & 2,500,000 \\ X_3 & \leq & 1,500,000 \\ X_4 & \leq & 1,800,000 \\ X_3 + X_4 & \geq & 0.55(X_1 + X_2 + X_3 + X_4) \\ X_1 & \geq & 0.15(X_1 + X_2 + X_3 + X_4) \\ X_1 + X_2 + X_3 + X_4 & \leq & 5,000,000 \\ X_1, X_2, X_3, X_4 & \geq & 0 \end{array}$$

Transportation Applications

■ Shipping Problem

- The transportation or shipping problem involves determining the amount of goods or items to be transported from a number of origins to a number of destinations
- The objective usually is to minimize total shipping costs or distances
- This is a specific case of LP and a special algorithm has been developed to solve it

Financial Applications

- The optimal solution to the ICT is to make the following investments

$$X_1 = \$750,000$$

$$X_2 = \$950,000$$

$$X_3 = \$1,500,000$$

$$X_4 = \$1,800,000$$

- The total interest earned with this plan is \$712,000

Transportation Applications

- The Top Speed Bicycle Co. manufactures and markets a line of 10-speed bicycles
- The firm has final assembly plants in two cities where labor costs are low
- It has three major warehouses near large markets
- The sales requirements for the next year are
 - New York – 10,000 bicycles
 - Chicago – 8,000 bicycles
 - Los Angeles – 15,000 bicycles
- The factory capacities are
 - New Orleans – 20,000 bicycles
 - Omaha – 15,000 bicycles

Transportation Applications

- The cost of shipping bicycles from the plants to the warehouses is different for each plant and warehouse

FROM \ TO	NEW YORK	CHICAGO	LOS ANGELES
New Orleans	\$2	\$3	\$5
Omaha	\$3	\$1	\$4

- The company wants to develop a shipping schedule that will minimize its total annual cost

Transportation Applications

- Objective function

Minimize total shipping costs

$$= 2X_{11} + 3X_{12} + 5X_{13} + 3X_{21} + 1X_{22} + 4X_{23}$$

subject to

$$X_{11} + X_{21} = 10,000 \quad (\text{New York demand})$$

$$X_{12} + X_{22} = 8,000 \quad (\text{Chicago demand})$$

$$X_{13} + X_{23} = 15,000 \quad (\text{Los Angeles demand})$$

$$X_{11} + X_{12} + X_{13} \leq 20,000 \quad (\text{New Orleans factory supply})$$

$$X_{21} + X_{22} + X_{23} \leq 15,000 \quad (\text{Omaha factory supply})$$

$$\text{All variables} \geq 0$$

Transportation Applications

- The double subscript variables will represent the origin factory and the destination warehouse

X_{ij} = bicycles shipped from factory i to warehouse j

- So

X_{11} = number of bicycles shipped from New Orleans to New York
 X_{12} = number of bicycles shipped from New Orleans to Chicago
 X_{13} = number of bicycles shipped from New Orleans to Los Angeles
 X_{21} = number of bicycles shipped from Omaha to New York
 X_{22} = number of bicycles shipped from Omaha to Chicago
 X_{23} = number of bicycles shipped from Omaha to Los Angeles

Transportation Applications

- Formulation for Excel's Solver

Enter the origin and destination names, the shipping costs, and the total supply and demand figures.

Our target cell is the total cost cell (B21), which we wish to minimize by changing the shipment cells (B17 through D18).

Enter the Solver, SOLVE on the menu bar at the top. If SOLVER is not a menu option in the Tools menu then g

These guarantee that we meet the demand exactly (3 constraints).

These guarantee that we do not exceed the supply (2 constraints).

Solver will place the shipments in these cells.

The total shipments to and from each location are calculated here.

The total cost is computed here by multiplying the unit shipping costs in the data table by the shipments in the shipment table using the SUMPRODUCT function. We are multiplying corresponding elements in tables this time rather than just rows or columns.

Program 8.5A

Transportation Applications

■ Solution from Excel's Solver

	A	B	C	D	E	F	G	H
1	Top Speed Bicycle Company							
2								
3	Transportation							
4	Enter the transportation costs, supplies and demands in the shaded area. Then go to							
5	TOOLS, SOLVER, SOLVE on the menu bar at the top.							
6	If SOLVER is not a menu option in the Tools menu then go to TOOLS, ADD-INS.							
7								
8	Data							
9	COSTS	New York	Chicago	Los Angeles	Supply			
10	New Orleans	2	3	5	20000			
11	Omaha	3	1	4	15000			
12	Demand	10000	8000	15000	33000 \ 35000			
13								
14								
15	Shipments							
16	Shipments	New York	Chicago	Los Angeles	Row Total			
17	New Orleans	10000	0	8000	18000			
18	Omaha	0	8000	7000	15000			
19	Column Total	10000	8000	15000	33000 \ 33000			
20								
21	Total Cost	96000						

Program 8.5A

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Transportation Applications

■ Truck Loading Problem

- The truck loading problem involves deciding which items to load on a truck so as to maximize the value of a load shipped
- Goodman Shipping has to ship the following six items

ITEM	VALUE (\$)	WEIGHT (POUNDS)
1	22,500	7,500
2	24,000	7,500
3	8,000	3,000
4	9,500	3,500
5	11,500	4,000
6	9,750	3,500

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Transportation Applications

■ Top Speed Bicycle solution

FROM \ TO			
	NEW YORK	CHICAGO	LOS ANGELES
New Orleans	10,000	0	8,000
Omaha	0	8,000	7,000

- Total shipping cost equals \$96,000
- Transportation problems are a special case of LP as the coefficients for every variable in the constraint equations equal 1
- This situation exists in assignment problems as well as they are a special case of the transportation problem

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Transportation Applications

- The objective is to maximize the value of items loaded into the truck
- The truck has a capacity of 10,000 pounds
- The decision variable is
 X_i = proportion of each item i loaded on the truck

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Transportation Applications

■ Objective function

$$\begin{aligned} \text{Maximize load value} &= \$22,500X_1 + \$24,000X_2 + \$8,000X_3 \\ &+ \$9,500X_4 + \$11,500X_5 + \$9,750X_6 \end{aligned}$$

subject to

$$\begin{aligned} &7,500X_1 + 7,500X_2 + 3,000X_3 \\ &+ 3,500X_4 + 4,000X_5 + 3,500X_6 \leq 10,000 \text{ lb capacity} \end{aligned}$$

$$X_1 \leq 1$$

$$X_2 \leq 1$$

$$X_3 \leq 1$$

$$X_4 \leq 1$$

$$X_5 \leq 1$$

$$X_6 \leq 1$$

$$X_1, X_2, X_3, X_4, X_5, X_6 \geq 0$$

Transportation Applications

■ Solver solution for Goodman Shipping

	A	B	C	D	E	F	G	H	I
1	Goodman Shipping								
2									
3	Item	Percent Loaded	Max Percent Loaded	Value (\$)	Weight (lbs)				
4	1	0.333333	1	22500	7500				
5	2	1	1	24000	7500				
6	3	0	1	8000	3000				
7	4	0	1	9500	3500				
8	5	0	1	11500	4000				
9	6	0	1	9750	3500				
10									
11			Total	\$ 31,500	10000				
12									
13			Weight Capacity		10000				

Program 8.6B

Transportation Applications

■ Excel Solver formulation for Goodman Shipping

The image shows an Excel spreadsheet for the Goodman Shipping problem. The data table is as follows:

Item	Percent loaded	Max percent loaded	Value (\$)	Weight (lbs)
1	0.5	1	22500	7500
2	0.5	1	24000	7500
3	0.5	1	8000	3000
4	0.5	1	9500	3500
5	0.5	1	11500	4000
6	0.5	1	9750	3500

The Solver Parameters dialog box is open, showing the following settings:

- Set Target Cell: $\$D\11
- Equal To: ☒ Max ☐ Min ☐ Value of: 0
- By Changing Variable Cells: $\$B\$4:\$B\9
- Subject to the Constraints:
 - $\$B\$4:\$B\$9 \leq \$C\$4:\$C\9
 - $\$E\$11 \leq \$E\13

Annotations in the image include:

- "Place the solution values here." pointing to the Percent loaded cells (B4:B9).
- "Enter data on value and weight here." pointing to the Value (\$) and Weight (lbs) columns.
- "The objective is the SUMPRODUCT of the item total value multiplied by the percent loaded." pointing to the formula in D11: $=\text{SUMPRODUCT}(\$B\$4:\$B\$9, D4:D9)$.
- "Do not load more than 100% of each item (6 constraints)." pointing to the constraints B4:B9 ≤ C4:C9.
- "Do not exceed 10,000 total pounds." pointing to the constraint E11 ≤ E13.

Program 8.6A

Transportation Applications

- The Goodman Shipping problem has an interesting issue
- The solution calls for one third of Item 1 to be loaded on the truck
- What if Item 1 can not be divided into smaller pieces?
- Rounding down leaves unused capacity on the truck and results in a value of \$24,000
- Rounding up is not possible since this would exceed the capacity of the truck
- Using **integer programming**, the solution is to load one unit of Items 3, 4, and 6 for a value of \$27,250

Transshipment Applications

- The transportation problem is a special case of the transshipment problem
- When the items are being moved from a source to a destination through an intermediate point (a **transshipment point**), the problem is called a **transshipment problem**

Transshipment Applications

■ Frosty Machines network

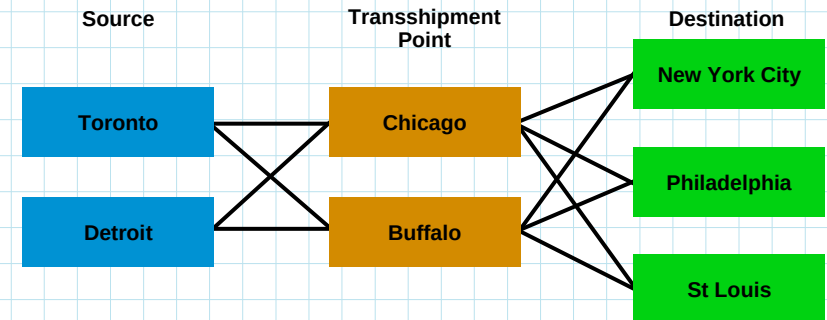


Figure 8.1

Transshipment Applications

■ Distribution Centers

- Frosty Machines manufactures snowblowers in Toronto and Detroit
- These are shipped to regional distribution centers in Chicago and Buffalo
- From there they are shipped to supply houses in New York, Philadelphia, and St Louis
- Shipping costs vary by location and destination
- Snowblowers can not be shipped directly from the factories to the supply houses

Transshipment Applications

■ Frosty Machines data

FROM	TO					SUPPLY
	CHICAGO	BUFFALO	NEW YORK CITY	PHILADELPHIA	ST LOUIS	
Toronto	\$4	\$7	—	—	—	800
Detroit	\$5	\$7	—	—	—	700
Chicago	—	—	\$6	\$4	\$5	—
Buffalo	—	—	\$2	\$3	\$4	—
Demand	—	—	450	350	300	

- Frosty would like to minimize the transportation costs associated with shipping snowblowers to meet the demands at the supply centers given the supplies available

Transshipment Applications

- A description of the problem would be to minimize cost subject to
 1. The number of units shipped from Toronto is not more than 800
 2. The number of units shipped from Detroit is not more than 700
 3. The number of units shipped to New York is 450
 4. The number of units shipped to Philadelphia is 350
 5. The number of units shipped to St Louis is 300
 6. The number of units shipped out of Chicago is equal to the number of units shipped into Chicago
 7. The number of units shipped out of Buffalo is equal to the number of units shipped into Buffalo

Transshipment Applications

- The linear program is

Minimize cost =

$$4T_1 + 7T_2 + 5D_1 + 7D_2 + 6C_1 + 4C_2 + 5C_3 + 2B_1 + 3B_2 + 4B_3$$

subject to

$$\begin{array}{ll} T_1 + T_2 \leq 800 & \text{(supply at Toronto)} \\ D_1 + D_2 \leq 700 & \text{(supply at Detroit)} \\ C_1 + B_1 = 450 & \text{(demand at New York)} \\ C_2 + B_2 = 350 & \text{(demand at Philadelphia)} \\ C_3 + B_3 = 300 & \text{(demand at St Louis)} \\ T_1 + D_1 = C_1 + C_2 + C_3 & \text{(shipping through Chicago)} \\ T_2 + D_2 = B_1 + B_2 + B_3 & \text{(shipping through Buffalo)} \\ T_1, T_2, D_1, D_2, C_1, C_2, C_3, B_1, B_2, B_3 \geq 0 & \text{(nonnegativity)} \end{array}$$

Transshipment Applications

- The decision variables should represent the number of units shipped from each source to the transshipment points and from there to the final destinations

T_1 = the number of units shipped from Toronto to Chicago
 T_2 = the number of units shipped from Toronto to Buffalo
 D_1 = the number of units shipped from Detroit to Chicago
 D_2 = the number of units shipped from Detroit to Buffalo
 C_1 = the number of units shipped from Chicago to New York
 C_2 = the number of units shipped from Chicago to Philadelphia
 C_3 = the number of units shipped from Chicago to St Louis
 B_1 = the number of units shipped from Buffalo to New York
 B_2 = the number of units shipped from Buffalo to Philadelphia
 B_3 = the number of units shipped from Buffalo to St Louis

Transshipment Applications

- The solution from QM for Windows is

Frosty Machines Transshipment Problem Solution												
	T1	T2	D1	D2	C1	C2	C3	B1	B2	B3	RHS	Dual
Minimize	4.	7.	5.	7.	6.	4.	5.	2.	3.	4.		
Toronto supply	1.	1.	0.	0.	0.	0.	0.	0.	0.	0.	<= 800.	0.
Detroit supply	0.	0.	1.	1.	0.	0.	0.	0.	0.	0.	<= 700.	0.
NY demand	0.	0.	0.	0.	1.	0.	0.	1.	0.	0.	= 450.	-9.
Philadelphia demand	0.	0.	0.	0.	0.	1.	0.	0.	1.	0.	= 350.	-8.
St. Louis demand	0.	0.	0.	0.	0.	0.	1.	0.	0.	1.	= 300.	-9.
Chicago	1.	0.	1.	0.	-1.	-1.	-1.	0.	0.	0.	= 0.	-4.
Buffalo	0.	1.	0.	1.	0.	0.	0.	-1.	-1.	-1.	= 0.	-7.
Solution->	650.	150.	0.	300.	0.	350.	300.	450.	0.	0.	9,550.	

Program 8.7

Ingredient Blending Applications

- **Diet Problems**
 - One of the earliest LP applications
 - Used to determine the most economical diet for hospital patients
 - Also known as the feed mix problem

Ingredient Blending Applications

- **We let**
 - X_A = pounds of grain A in one 2-ounce serving of cereal
 - X_B = pounds of grain B in one 2-ounce serving of cereal
 - X_C = pounds of grain C in one 2-ounce serving of cereal

- **Whole Foods Natural Cereal requirements**

GRAIN	COST PER POUND (CENTS)	PROTEIN (UNITS/LB)	RIBOFLAVIN (UNITS/LB)	PHOSPHOROUS (UNITS/LB)	MAGNESIUM (UNITS/LB)
A	33	22	16	8	5
B	47	28	14	7	0
C	38	21	25	9	6

Table 8.5

Ingredient Blending Applications

- The Whole Food Nutrition Center uses three bulk grains to blend a natural cereal
- They advertise the cereal meets the U.S. Recommended Daily Allowance (USRDA) for four key nutrients
- They want to select the blend that will meet the requirements at the minimum cost

NUTRIENT	USRDA
Protein	3 units
Riboflavin	2 units
Phosphorus	1 unit
Magnesium	0.425 units

Ingredient Blending Applications

- **The objective function is**

Minimize total cost of mixing a 2-ounce serving = $\$0.33X_A + \$0.47X_B + \$0.38X_C$

subject to

$$\begin{aligned}
 22X_A + 28X_B + 21X_C &\geq 3 && \text{(protein units)} \\
 16X_A + 14X_B + 25X_C &\geq 2 && \text{(riboflavin units)} \\
 8X_A + 7X_B + 9X_C &\geq 1 && \text{(phosphorous units)} \\
 5X_A + 0X_B + 6X_C &\geq 0.425 && \text{(magnesium units)} \\
 X_A + X_B + X_C &= 0.125 && \text{(total mix)} \\
 X_A, X_B, X_C &\geq 0
 \end{aligned}$$

Ingredient Blending Applications

■ Whole Food solution using QM for Windows

Linear Programming Results						
Whole Food Nutrition Center Solution						
	X1	X2	X3		RHS	Dual
Minimize	0.33	0.47	0.38			
Constraint 1	22.	28.	21.	>=	3.	-0.038
Constraint 2	16.	14.	25.	>=	2.	0.
Constraint 3	8.	7.	9.	>=	1.	-0.088
Constraint 4	5.	0.	6.	>=	0.425	0.
Constraint 5	1.	1.	1.	=	0.125	1.21
Solution->	0.025	0.05	0.05		0.05	

Program 8.8

Ingredient Blending Applications

- The Low Knock Oil Company produces two grades of cut-rate gasoline for industrial distribution
- The two grades, regular and economy, are created by blending two different types of crude oil
- The crude oil differs in cost and in its content of crucial ingredients

CRUDE OIL TYPE	INGREDIENT A (%)	INGREDIENT B (%)	COST/BARREL (\$)
X100	35	55	30.00
X220	60	25	34.80

Ingredient Blending Applications

■ Ingredient Mix and Blending Problems

- Diet and feed mix problems are special cases of a more general class of problems known as **ingredient or blending problems**
- Blending problems arise when decisions must be made regarding the blending of two or more resources to produce one or more product
- Resources may contain essential ingredients that must be blended so that a specified percentage is in the final mix

Ingredient Blending Applications

■ The firm lets

- X_1 = barrels of crude X100 blended to produce the refined regular
- X_2 = barrels of crude X100 blended to produce the refined economy
- X_3 = barrels of crude X220 blended to produce the refined regular
- X_4 = barrels of crude X220 blended to produce the refined economy

■ The objective function is

$$\text{Minimize cost} = \$30X_1 + \$30X_2 + \$34.80X_3 + \$34.80X_4$$

Ingredient Blending Applications

■ Problem formulation

At least 45% of each barrel of regular must be ingredient A

$(X_1 + X_3)$ = total amount of crude blended to produce the refined regular gasoline demand

Thus,

$0.45(X_1 + X_3)$ = amount of ingredient A required

But

$0.35X_1 + 0.60X_3$ = amount of ingredient A in refined regular gas

So

$$0.35X_1 + 0.60X_3 \geq 0.45X_1 + 0.45X_3$$

or

$$-0.10X_1 + 0.15X_3 \geq 0 \quad (\text{ingredient A in regular constraint})$$

Ingredient Blending Applications

■ Solution from QM for Windows

Low Knock Oil Company Solution						
	X1	X2	X3	X4	RHS	Dual
Minimize	30.	30.	34.8	34.8		
Constraint 1	1.	0.	1.	0.	>= 25,000.	-31.92
Constraint 2	0.	1.	0.	1.	>= 32,000.	-30.8
Constraint 3	-0.1	0.	0.15	0.	>= 0.	-19.2
Constraint 4	0.	0.05	0.	-0.25	<= 0.	16.
Solution->	15,000.	26,666.67	10,000.	5,333.333		1,783,599.99

Program 8.9

Ingredient Blending Applications

■ Problem formulation

Minimize cost = $30X_1 + 30X_2 + 34.80X_3 + 34.80X_4$

subject to $X_1 + X_3 \geq 25,000$

$X_2 + X_4 \geq 32,000$

$-0.10X_1 + 0.15X_3 \geq 0$

$0.05X_2 - 0.25X_4 \leq 0$

$X_1, X_2, X_3, X_4 \geq 0$