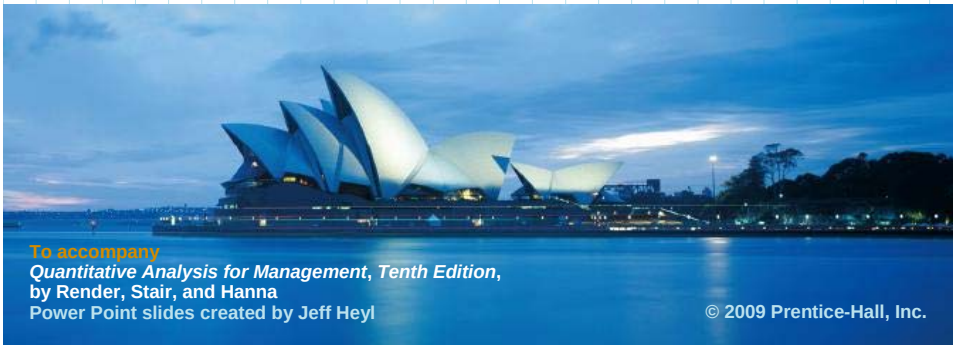


Chapter 7

Linear Programming Models: Graphical and Computer Methods



Learning Objectives

After completing this chapter, students will be able to:

1. Understand the basic assumptions and properties of linear programming (LP)
2. Graphically solve any LP problem that has only two variables by both the corner point and isoprofit line methods
3. Understand special issues in LP such as infeasibility, unboundedness, redundancy, and alternative optimal solutions
4. Understand the role of sensitivity analysis
5. Use Excel spreadsheets to solve LP problems

Chapter Outline

- 7.1 Introduction
- 7.2 Requirements of a Linear Programming Problem
- 7.3 Formulating LP Problems
- 7.4 Graphical Solution to an LP Problem
- 7.5 Solving Flair Furniture's LP Problem using QM for Windows and Excel
- 7.6 Solving Minimization Problems
- 7.7 Four Special Cases in LP
- 7.8 Sensitivity Analysis

Introduction

- Many management decisions involve trying to make the most effective use of limited resources
 - Machinery, labor, money, time, warehouse space, raw materials
- **Linear programming (LP)** is a widely used mathematical modeling technique designed to help managers in planning and decision making relative to resource allocation
- Belongs to the broader field of **mathematical programming**
- In this sense, **programming** refers to modeling and solving a problem mathematically

Requirements of a Linear Programming Problem

- LP has been applied in many areas over the past 50 years
- All LP problems have 4 properties in common
 1. All problems seek to **maximize** or **minimize** some quantity (the **objective function**)
 2. The presence of restrictions or **constraints** that limit the degree to which we can pursue our objective
 3. There must be alternative courses of action to choose from
 4. The objective and constraints in problems must be expressed in terms of **linear** equations or **inequalities**

Basic Assumptions of LP

- We assume conditions of **certainty** exist and numbers in the objective and constraints are known with certainty and do not change during the period being studied
- We assume **proportionality** exists in the objective and constraints
- We assume **additivity** in that the total of all activities equals the sum of the individual activities
- We assume **divisibility** in that solutions need not be whole numbers
- All answers or variables are **nonnegative**

LP Properties and Assumptions

PROPERTIES OF LINEAR PROGRAMS

1. One objective function
2. One or more constraints
3. Alternative courses of action
4. Objective function and constraints are linear

ASSUMPTIONS OF LP

1. Certainty
2. Proportionality
3. Additivity
4. Divisibility
5. Nonnegative variables

Table 7.1

Formulating LP Problems

- Formulating a linear program involves developing a mathematical model to represent the managerial problem
- The steps in formulating a linear program are
 1. Completely understand the managerial problem being faced
 2. Identify the objective and constraints
 3. Define the decision variables
 4. Use the decision variables to write mathematical expressions for the objective function and the constraints

Formulating LP Problems

- One of the most common LP applications is the **product mix problem**
- Two or more products are produced using limited resources such as personnel, machines, and raw materials
- The profit that the firm seeks to maximize is based on the profit contribution per unit of each product
- The company would like to determine how many units of each product it should produce so as to maximize overall profit given its limited resources

Flair Furniture Company

- The company wants to determine the best combination of tables and chairs to produce to reach the maximum profit

DEPARTMENT	HOURS REQUIRED TO PRODUCE 1 UNIT		AVAILABLE HOURS THIS WEEK
	(T) TABLES	(C) CHAIRS	
Carpentry	4	3	240
Painting and varnishing	2	1	100
Profit per unit	\$70	\$50	

Table 7.2

Flair Furniture Company

- The Flair Furniture Company produces inexpensive tables and chairs
- Processes are similar in that both require a certain amount of hours of carpentry work and in the painting and varnishing department
- Each table takes 4 hours of carpentry and 2 hours of painting and varnishing
- Each chair requires 3 of carpentry and 1 hour of painting and varnishing
- There are 240 hours of carpentry time available and 100 hours of painting and varnishing
- Each table yields a profit of \$70 and each chair a profit of \$50

Flair Furniture Company

- The objective is to
Maximize profit
- The constraints are
 1. The hours of carpentry time used cannot exceed 240 hours per week
 2. The hours of painting and varnishing time used cannot exceed 100 hours per week
- The decision variables representing the actual decisions we will make are
 T = number of tables to be produced per week
 C = number of chairs to be produced per week

Flair Furniture Company

- We create the LP objective function in terms of T and C

$$\text{Maximize profit} = \$70T + \$50C$$

- Develop mathematical relationships for the two constraints
- For carpentry, total time used is
(4 hours per table)(Number of tables produced)
+ (3 hours per chair)(Number of chairs produced)
- We know that
Carpentry time used $\leq$ Carpentry time available
 $4T + 3C \leq 240$ (hours of carpentry time)

Flair Furniture Company

- The values for T and C must be nonnegative

$$T \geq 0 \text{ (number of tables produced is greater than or equal to 0)}$$

$$C \geq 0 \text{ (number of chairs produced is greater than or equal to 0)}$$

- The complete problem stated mathematically

$$\text{Maximize profit} = \$70T + \$50C$$

subject to

$$4T + 3C \leq 240 \text{ (carpentry constraint)}$$

$$2T + 1C \leq 100 \text{ (painting and varnishing constraint)}$$

$$T, C \geq 0 \text{ (nonnegativity constraint)}$$

Flair Furniture Company

- Similarly
Painting and varnishing time used
 $\leq$ Painting and varnishing time available
2 $T + 1C \leq 100$ (hours of painting and varnishing time)

This means that each table produced requires two hours of painting and varnishing time

- Both of these constraints restrict production capacity and affect total profit

Graphical Solution to an LP Problem

- The easiest way to solve a small LP problems is with the graphical solution approach
- The graphical method only works when there are just two decision variables
- When there are more than two variables, a more complex approach is needed as it is not possible to plot the solution on a two-dimensional graph
- The graphical method provides valuable insight into how other approaches work

Graphical Representation of a Constraint

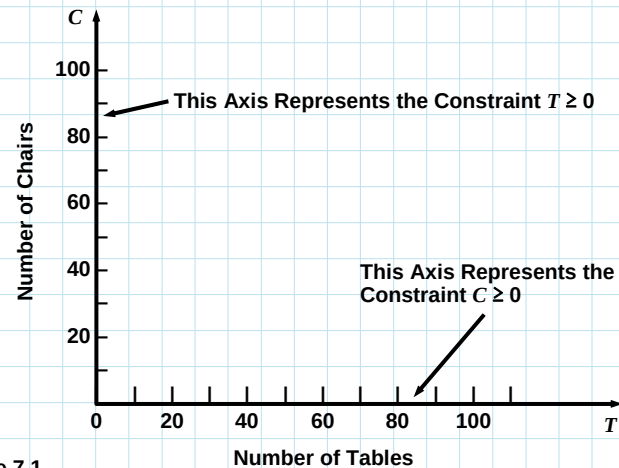


Figure 7.1

Graphical Representation of a Constraint

- When Flair produces no tables, the carpentry constraint is

$$4(0) + 3C = 240$$

$$3C = 240$$

$$C = 80$$

- Similarly for no chairs

$$4T + 3(0) = 240$$

$$4T = 240$$

$$T = 60$$

- This line is shown on the following graph

Graphical Representation of a Constraint

- The first step in solving the problem is to identify a set or region of feasible solutions
- To do this we plot each constraint equation on a graph
- We start by graphing the equality portion of the constraint equations

$$4T + 3C = 240$$
- We solve for the axis intercepts and draw the line

Graphical Representation of a Constraint

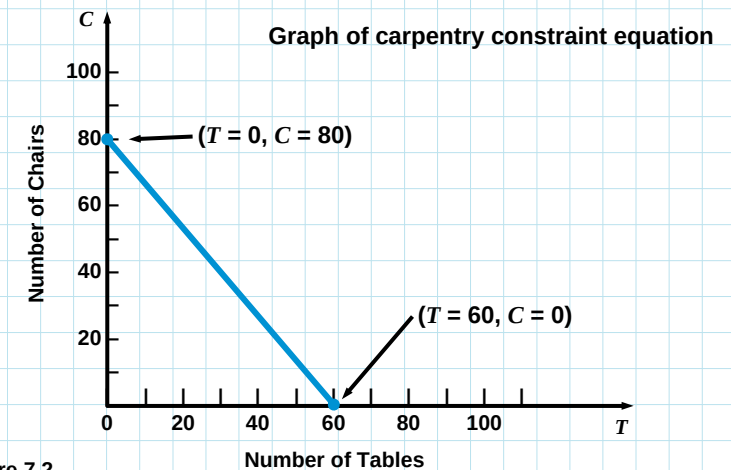


Figure 7.2

Graphical Representation of a Constraint

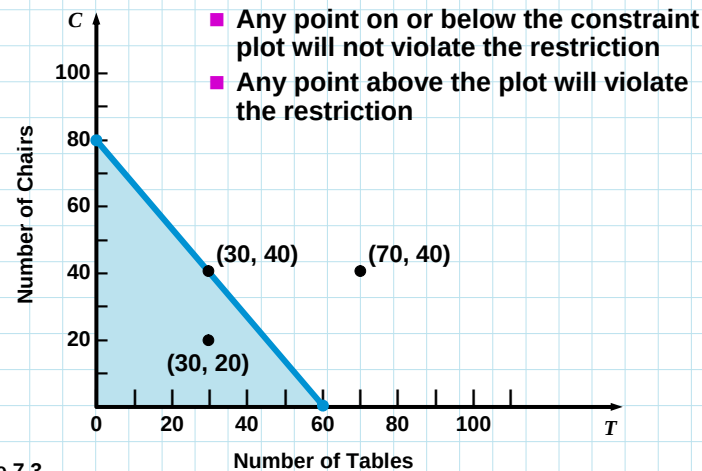


Figure 7.3

Graphical Representation of a Constraint

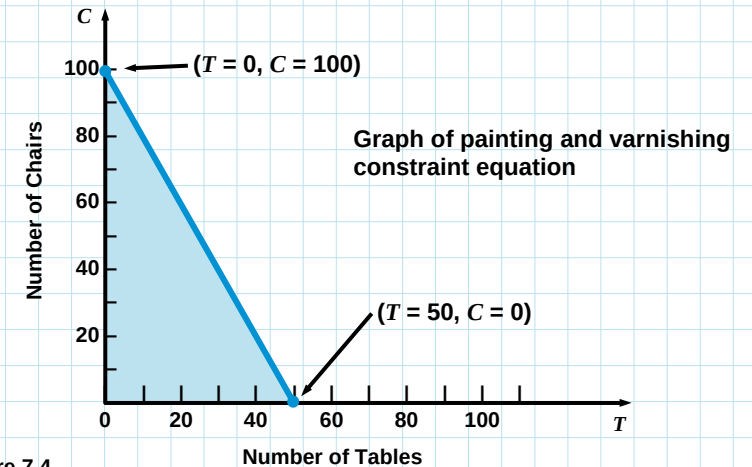


Figure 7.4

Graphical Representation of a Constraint

- The point (30, 40) lies on the plot and exactly satisfies the constraint

$$4(30) + 3(40) = 240$$
- The point (30, 20) lies below the plot and satisfies the constraint

$$4(30) + 3(20) = 180$$
- The point (70, 40) lies above the plot and does not satisfy the constraint

$$4(70) + 3(40) = 400$$

Graphical Representation of a Constraint

- To produce tables and chairs, both departments must be used
- We need to find a solution that satisfies both constraints *simultaneously*
- A new graph shows both constraint plots
- The *feasible region* (or *area of feasible solutions*) is where all constraints are satisfied
- Any point inside this region is a *feasible* solution
- Any point outside the region is an *infeasible* solution

Graphical Representation of a Constraint

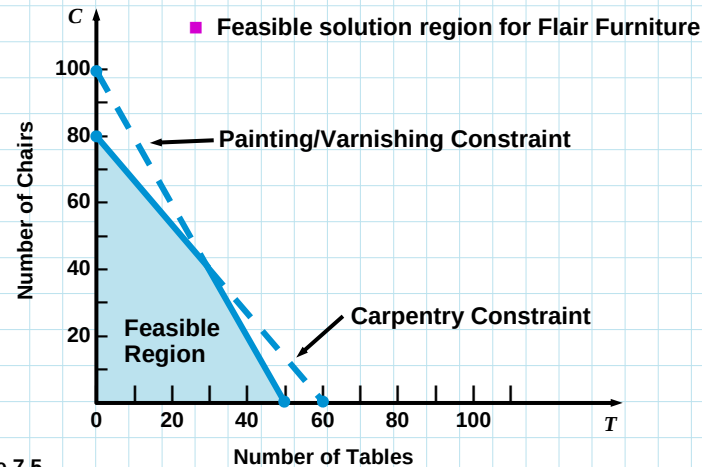


Figure 7.5

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Graphical Representation of a Constraint

■ For the point (50, 5)

Carpentry constraint	$4T + 3C \leq 240$ hours available $(4)(50) + (3)(5) = 215$ hours used	✓
Painting constraint	$2T + 1C \leq 100$ hours available $(2)(50) + (1)(5) = 105$ hours used	✗

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Graphical Representation of a Constraint

■ For the point (30, 20)

Carpentry constraint	$4T + 3C \leq 240$ hours available $(4)(30) + (3)(20) = 180$ hours used	✓
Painting constraint	$2T + 1C \leq 100$ hours available $(2)(30) + (1)(20) = 80$ hours used	✓

■ For the point (70, 40)

Carpentry constraint	$4T + 3C \leq 240$ hours available $(4)(70) + (3)(40) = 400$ hours used	✗
Painting constraint	$2T + 1C \leq 100$ hours available $(2)(70) + (1)(40) = 180$ hours used	✗

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Isoprofit Line Solution Method

- Once the feasible region has been graphed, we need to find the optimal solution from the many possible solutions
- The speediest way to do this is to use the isoprofit line method
- Starting with a small but possible profit value, we graph the objective function
- We move the objective function line in the direction of increasing profit while maintaining the slope
- The last point it touches in the feasible region is the optimal solution

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Isoprofit Line Solution Method

- For Flair Furniture, choose a profit of \$2,100
- The objective function is then

$$\$2,100 = 70T + 50C$$
- Solving for the axis intercepts, we can draw the graph
- This is obviously not the best possible solution
- Further graphs can be created using larger profits
- The further we move from the origin, the larger the profit will be
- The highest profit (\$4,100) will be generated when the isoprofit line passes through the point (30, 40)

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Isoprofit Line Solution Method

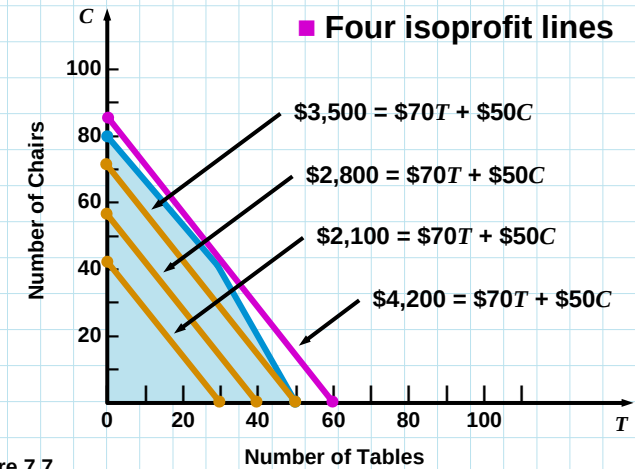


Figure 7.7

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Isoprofit Line Solution Method

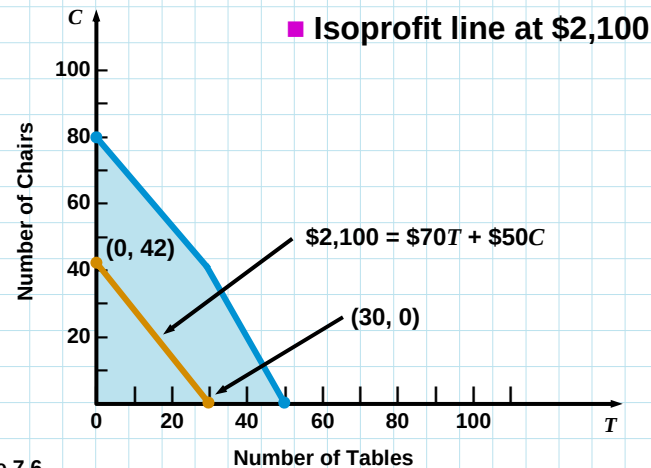


Figure 7.6

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Isoprofit Line Solution Method

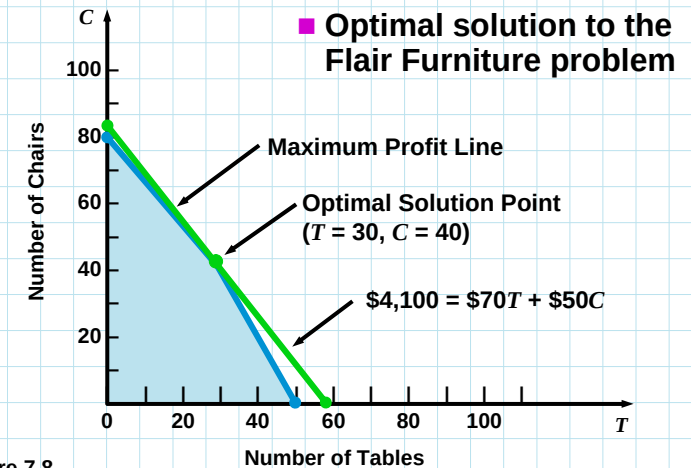


Figure 7.8

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Corner Point Solution Method

- A second approach to solving LP problems employs the **corner point method**
- It involves looking at the profit at every corner point of the feasible region
- The mathematical theory behind LP is that the optimal solution must lie at one of the **corner points**, or **extreme point**, in the feasible region
- For Flair Furniture, the feasible region is a four-sided polygon with four corner points labeled 1, 2, 3, and 4 on the graph

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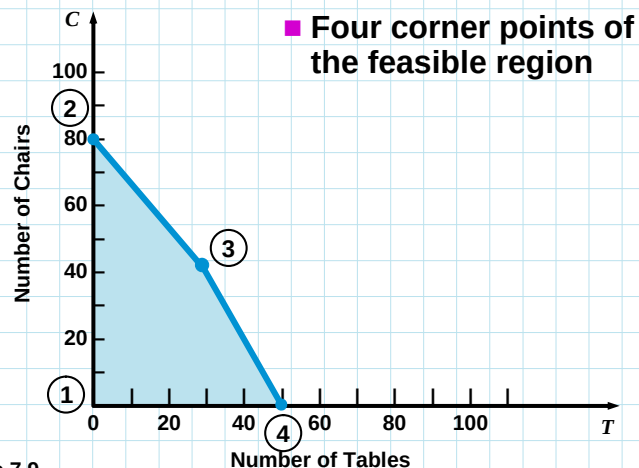
Corner Point Solution Method

Point ①: ($T = 0, C = 0$)	Profit = $\$70(0) + \$50(0) = \$0$
Point ②: ($T = 0, C = 80$)	Profit = $\$70(0) + \$50(80) = \$4,000$
Point ④: ($T = 50, C = 0$)	Profit = $\$70(50) + \$50(0) = \$3,500$
Point ③: ($T = 30, C = 40$)	Profit = $\$70(30) + \$50(40) = \$4,100$

- Because Point ③ returns the highest profit, this is the optimal solution
- To find the coordinates for Point ③ accurately we have to solve for the intersection of the two constraint lines
- The details of this are on the following slide

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Corner Point Solution Method



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Corner Point Solution Method

- Using the **simultaneous equations method**, we multiply the painting equation by -2 and add it to the carpentry equation

$$\begin{array}{rcl}
 4T + 3C & = & 240 \quad \text{(carpentry line)} \\
 -4T - 2C & = & -200 \quad \text{(painting line)} \\
 \hline
 C & = & 40
 \end{array}$$

- Substituting 40 for C in either of the original equations allows us to determine the value of T

$$\begin{array}{rcl}
 4T + (3)(40) & = & 240 \quad \text{(carpentry line)} \\
 4T + 120 & = & 240 \\
 T & = & 30
 \end{array}$$

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Summary of Graphical Solution Methods

ISOPROFIT METHOD

1. Graph all constraints and find the feasible region.
2. Select a specific profit (or cost) line and graph it to find the slope.
3. Move the objective function line in the direction of increasing profit (or decreasing cost) while maintaining the slope. The last point it touches in the feasible region is the optimal solution.
4. Find the values of the decision variables at this last point and compute the profit (or cost).

CORNER POINT METHOD

1. Graph all constraints and find the feasible region.
2. Find the corner points of the feasible reason.
3. Compute the profit (or cost) at each of the feasible corner points.
4. select the corner point with the best value of the objective function found in Step 3. This is the optimal solution.

Table 7.3

Using QM for Windows

- First select the Linear Programming module
- Specify the number of constraints (non-negativity is assumed)
- Specify the number of decision variables
- Specify whether the objective is to be maximized or minimized
- For the Flair Furniture problem there are two constraints, two decision variables, and the objective is to maximize profit

Solving Flair Furniture's LP Problem Using QM for Windows and Excel

- Most organizations have access to software to solve big LP problems
- While there are differences between software implementations, the approach each takes towards handling LP is basically the same
- Once you are experienced in dealing with computerized LP algorithms, you can easily adjust to minor changes

Using QM for Windows

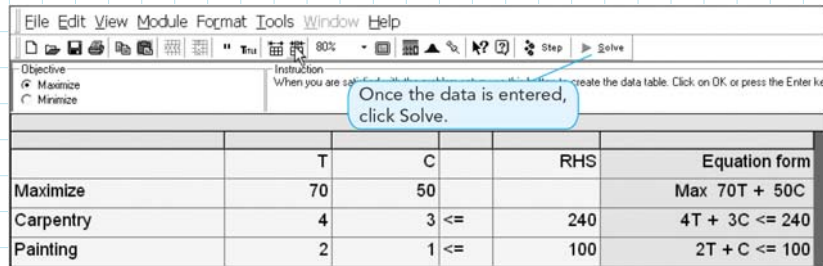
- Computer screen for input of data

	X1	X2	RHS	Equation form
Maximize	0	0		Max
Constraint 1	0	0	<= 0	<= 0
Constraint 2	0	0	<= 0	<= 0

Program 7.1A

Using QM for Windows

Computer screen for input of data

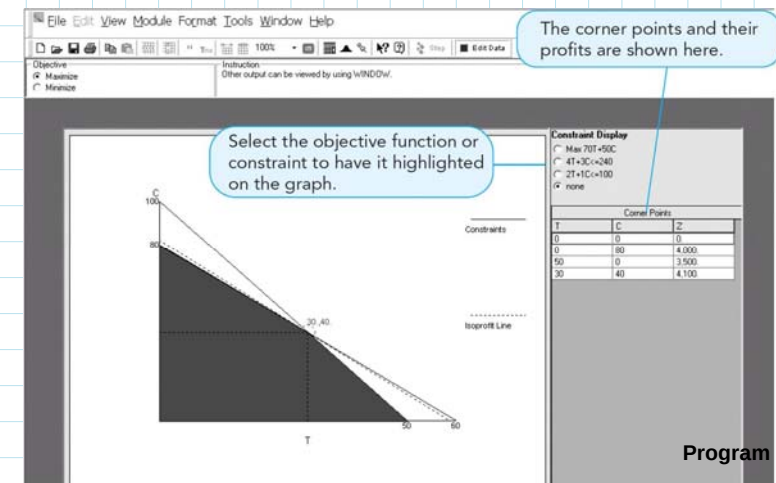


	T	C	RHS	Equation form
Maximize	70	50		Max 70T + 50C
Carpentry	4	3	≤ 240	4T + 3C ≤ 240
Painting	2	1	≤ 100	2T + C ≤ 100

Program 7.1B

Using QM for Windows

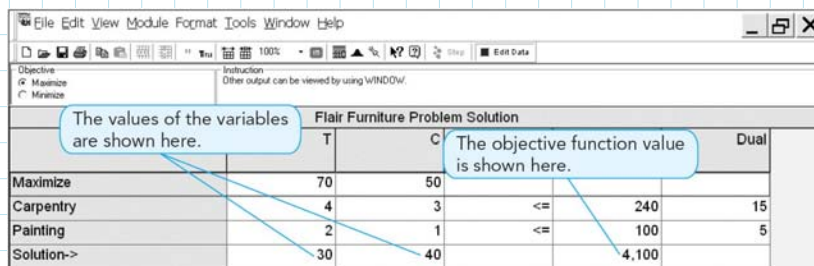
Graphical output of solution



Program 7.1D

Using QM for Windows

Computer screen for output of solution



	T	C	Dual
Maximize	70	50	
Carpentry	4	3	≤ 240
Painting	2	1	≤ 100
Solution->	30	40	4,100

Program 7.1C

Using Excel's Solver Command to Solve LP Problems

- The Solver tool in Excel can be used to find solutions to
 - LP problems
 - Integer programming problems
 - Noninteger programming problems
- Solver may be sensitive to the initial values it uses
- Solver is limited to 200 variables and 100 constraints
- It can be used for small real world problems
- Add-ins like What'sBest! Can be used to expand Solver's capabilities

Using Solver to Solve the Flair Furniture Problem

- Recall the model for Flair Furniture is

$$\begin{aligned} \text{Maximize profit} &= \$70T + \$50C \\ \text{Subject to} \quad &4T + 3C \leq 240 \\ &2T + 1C \leq 100 \end{aligned}$$

- To use Solver, it is necessary to enter formulas based on the initial model
- Program 7.2A shows how the data and formulas are entered into Solver to solve this problem
- The following slide details the steps in creating the Solver model

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Using Solver to Solve the Flair Furniture Problem

- Enter the variable names and the coefficients for the objective function and constraints
- Specify cells where the values of the variables will be located
- Write a formula to compute the value of the objective function
- Write formulas to compute the left-hand sides of the constraints
- Indicate constraint signs ($\leq$, $=$, and $\geq$) for display purposes only
- Input the right-hand side values for each constraint
- If desired, write a formula for the slack of each constraint

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Using Solver to Solve the Flair Furniture Problem

Program 7.2A

Enter the names for the objective function and constraints.

We will place the answers here. Using 1s as the initial data rather than 0s makes it easier to check the formulas entered in the spreadsheet.

The actual constraint signs are entered in Solver. These in column E are for display purposes only.

Calculate the total for the objective function and the constraints.

The objective is in cell D4, and it is to be maximized.

Calculate slack as the difference between the right-hand side and the left-hand side.

The cells to change (variables) are specified as B8 to C8.

Cells D5 through D6 (the left-hand side) must be less than cells E5 through E6 (the right-hand side) respectively. The single line represents two constraints. That is, D5 must be less than E5 and D6 must be less than E6.

Click here to get to the Options screen, which allows you to select a linear model and to assume that variables are nonnegative. (In Excel 5, the nonnegativity conditions must be entered explicitly as constraints.)

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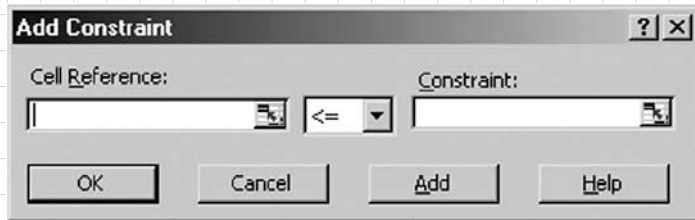
Using Solver to Solve the Flair Furniture Problem

- Once the model has been entered, the following steps can be used to solve the problem
- In Excel 2003, select **Tools—Solver**
 - In Excel 2007, select the **Data** tab and then select **Solver** in the **Analysis** group
- If Solver does not appear in the indicated place, follow the installation instructions in the text*
- Once Solver has been selected, a window will open for the input of the Solver Parameters. Move the cursor to the **Set Target Cell** box and fill in the cell that is used to calculate the value of the objective function.

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Using Solver to Solve the Flair Furniture Problem

3. Move the cursor to the **By Changing Cells** box and input the cells that will contain the values for the variables
4. Move the cursor to the **Subject to the Constraints** box, and then select **Add**



Program 7.2B

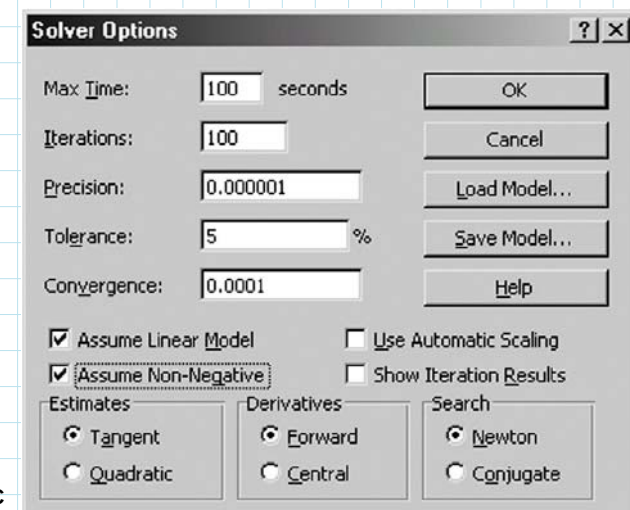
Using Solver to Solve the Flair Furniture Problem

8. In the **Solver Parameters** window, click **Options** and check **Assume Linear Model** and check **Assume Non-negative**, as illustrated in Program 7.2C, then click **OK**
9. Review the information in the Solver window to make sure it is correct, and click **Solve**
10. The **Solver Solutions** window in Program 7.2D is displayed and indicates that a solution was found. Select **Keep Solver Solution**, and the values in the spreadsheet will be kept at the optimal solution. You may select what sort of additional information (e.g., **Sensitivity**) is to be presented from the reports Window.

Using Solver to Solve the Flair Furniture Problem

5. The **Cell Reference** box is for the range of cells that contain the left-hand sides of the constraints
6. Click the $\leq$ to change the type of constraint if necessary (If there had been some $\geq$ or $=$ constraints in addition to the $\leq$ constraints, it would be best to input all of one type [e.g., $\leq$] first, and then select **Add** to input the other type of constraint)
7. Move the cursor to the **Constraint** box to input the right-hand sides of the constraints. Click **Add** to finish this set of constraints and begin inputting another set, or click **OK** if there are no other constraints to add.

Using Solver to Solve the Flair Furniture Problem



Program 7.2C

Using Solver to Solve the Flair Furniture Problem

	A	B	C	D	E	F	G
1	Flair Furniture						
2							
3		Tables	Chairs	Left Hand Side	Right Hand Side	Slack	
4	Objective function	70	50	4100			
5	Carpentry	4	3	240	<=	240	0
6	Painting	2	1	100	<=	100	0
7							
8	Solution Values	30	40				
9							
10							
11							
12							
13							
14							
15							
16							
17							
18							
19							
20							
21							

Solver has found that we should produce 30 tables and 40 chairs. The total profit is 4,100 in cell D4.

The answers are shown on the spreadsheet, but there are also more complete reports available.

It is important to check the statement made by Solver. In this case, it says that "Solver found a solution." In other problems, this may not be the case. For some problems, there may be no feasible solution, and for others, more iterations may be required.

Solver Results
 Solver found a solution. All constraints and optimality conditions are satisfied.
☒ Keep Solver Solution
☐ Restore Original Values
 Reports: Answer, Sensitivity, Limits
 OK Cancel Save Scenario... Help

Program 7.2D

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Solving Minimization Problems

- Many LP problems involve minimizing an objective such as cost instead of maximizing a profit function
- Minimization problems can be solved graphically by first setting up the feasible solution region and then using either the corner point method or an isocost line approach (which is analogous to the isoprofit approach in maximization problems) to find the values of the decision variables (e.g., X_1 and X_2) that yield the minimum cost

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Using Solver to Solve the Flair Furniture Problem

- Excel's answer report for the Flair Furniture problem

	A	B	C	D	E	F	G
1	Microsoft Excel 11.0 Answer Report						
2							
3							
4							
5							
6	Target Cell (Max)						
7	Cell	Name	Original Value	Final Value			
8	\$D\$4	Objective function Left Hand Side	120	4100			
9							
10							
11	Adjustable Cells						
12	Cell	Name	Original Value	Final Value			
13	\$B\$8	Solution Values Tables	1	30			
14	\$C\$8	Solution Values Chairs	1	40			
15							
16							
17	Constraints						
18	Cell	Name	Cell Value	Formula	Status	Slack	
19	\$D\$5	Carpentry Left Hand Side	240	\$D\$5<=\$F\$5	Binding	0	
20	\$D\$6	Painting Left Hand Side	100	\$D\$6<=\$F\$6	Binding	0	

The solution values and the initial values are displayed here.

The slack is shown here as well as on the data sheet.

This is the amount used.

Program 7.2E

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Holiday Meal Turkey Ranch

- The Holiday Meal Turkey Ranch is considering buying two different brands of turkey feed and blending them to provide a good, low-cost diet for its turkeys

Let

 X_1 = number of pounds of brand 1 feed purchased X_2 = number of pounds of brand 2 feed purchasedMinimize cost (in cents) = $2X_1 + 3X_2$

subject to:

 $5X_1 + 10X_2 \geq 90$ ounces (ingredient constraint A) $4X_1 + 3X_2 \geq 48$ ounces (ingredient constraint B) $0.5X_1 \geq 1.5$ ounces (ingredient constraint C) $X_1 \geq 0$ (nonnegativity constraint) $X_2 \geq 0$ (nonnegativity constraint)

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Holiday Meal Turkey Ranch

Holiday Meal Turkey Ranch data

INGREDIENT	COMPOSITION OF EACH POUND OF FEED (OZ.)		MINIMUM MONTHLY REQUIREMENT PER TURKEY (OZ.)
	BRAND 1 FEED	BRAND 2 FEED	
A	5	10	90
B	4	3	48
C	0.5	0	1.5
Cost per pound	2 cents	3 cents	

Table 7.4

Holiday Meal Turkey Ranch

- We solve for the values of the three corner points
- Point a is the intersection of ingredient constraints C and B

$$4X_1 + 3X_2 = 48$$

$$X_1 = 3$$

- Substituting 3 in the first equation, we find $X_2 = 12$
- Solving for point b with basic algebra we find $X_1 = 8.4$ and $X_2 = 4.8$
- Solving for point c we find $X_1 = 18$ and $X_2 = 0$

Holiday Meal Turkey Ranch

- Using the corner point method
- First we construct the feasible solution region
- The optimal solution will lie at one of the corners as it would in a maximization problem

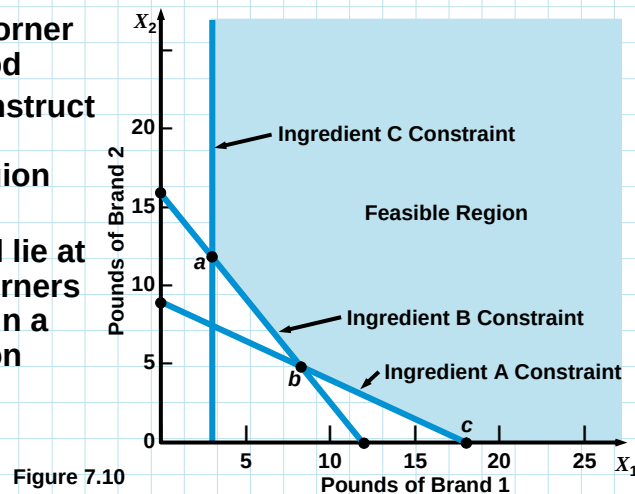


Figure 7.10

Holiday Meal Turkey Ranch

- Substituting these values back into the objective function we find

$$\text{Cost} = 2X_1 + 3X_2$$

$$\text{Cost at point a} = 2(3) + 3(12) = 42$$

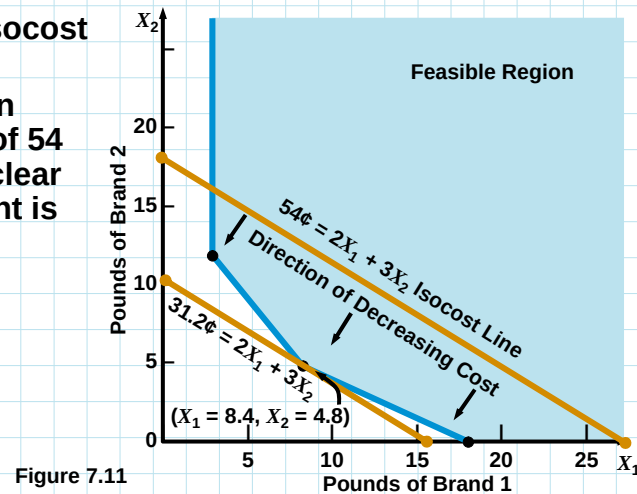
$$\text{Cost at point b} = 2(8.4) + 3(4.8) = 31.2$$

$$\text{Cost at point c} = 2(18) + 3(0) = 36$$

- The lowest cost solution is to purchase 8.4 pounds of brand 1 feed and 4.8 pounds of brand 2 feed for a total cost of 31.2 cents per turkey

Holiday Meal Turkey Ranch

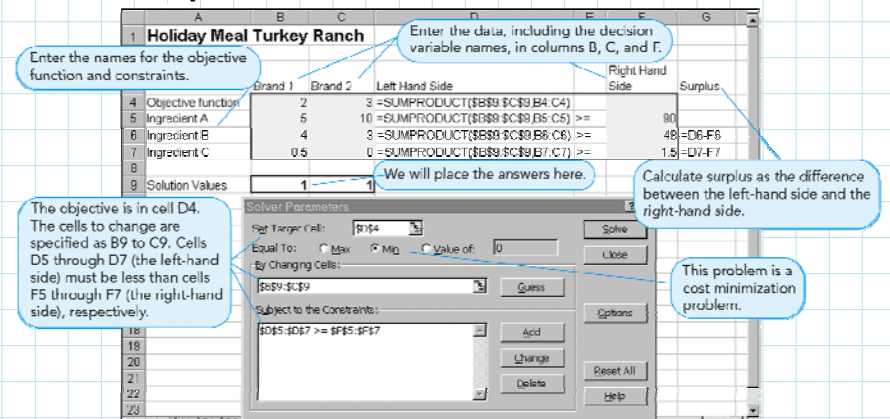
- Using the isocost approach
- Choosing an initial cost of 54 cents, it is clear improvement is possible



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Holiday Meal Turkey Ranch

- Setting up Solver to solve the Holiday Meal Turkey Ranch problem



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Holiday Meal Turkey Ranch

- QM for Windows can also be used to solve the Holiday Meal Turkey Ranch problem

QM for Windows - C:\Prentice\Data\RenderStair7\HolidayMeal.LIN

Objective: ☐ Maximize ☒ Minimize

Linear Programming Results

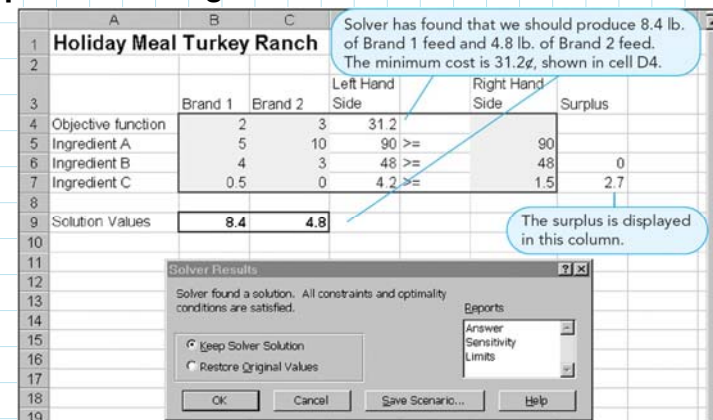
Holiday Meal Turkey Ranch Solution					
	Brand 1	Brand 2		RHS	Dual
Minimize	2.	3.			
Ingredient A	5.	10.	>=	90.	-0.24
Ingredient B	4.	3.	>=	48.	-0.2
Ingredient C	0.5	0.	>=	1.5	0.
Solution->	8.4	4.8		31.2	

Program 7.3

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Holiday Meal Turkey Ranch

- Solution to the Holiday Meal Turkey Ranch problem using Solver



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Four Special Cases in LP

- Four special cases and difficulties arise at times when using the graphical approach to solving LP problems
 - Infeasibility
 - Unboundedness
 - Redundancy
 - Alternate Optimal Solutions

Four Special Cases in LP

- A problem with no feasible solution

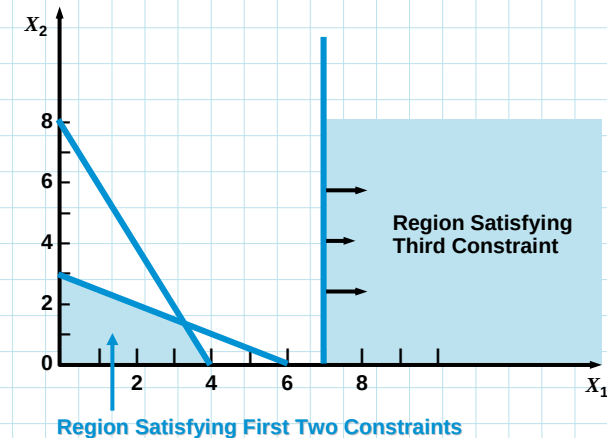


Figure 7.12

Four Special Cases in LP

- No feasible solution
 - Exists when there is no solution to the problem that satisfies all the constraint equations
 - No feasible solution region exists
 - This is a common occurrence in the real world
 - Generally one or more constraints are relaxed until a solution is found

Four Special Cases in LP

- Unboundedness
 - Sometimes a linear program will not have a finite solution
 - In a maximization problem, one or more solution variables, and the profit, can be made infinitely large without violating any constraints
 - In a graphical solution, the feasible region will be open ended
 - This usually means the problem has been formulated improperly

Four Special Cases in LP

- A solution region unbounded to the right

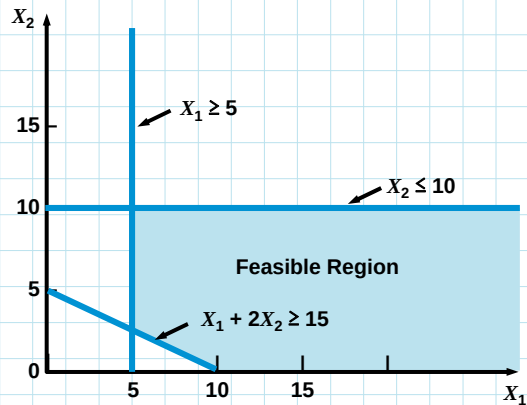


Figure 7.13

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Four Special Cases in LP

- A problem with a redundant constraint

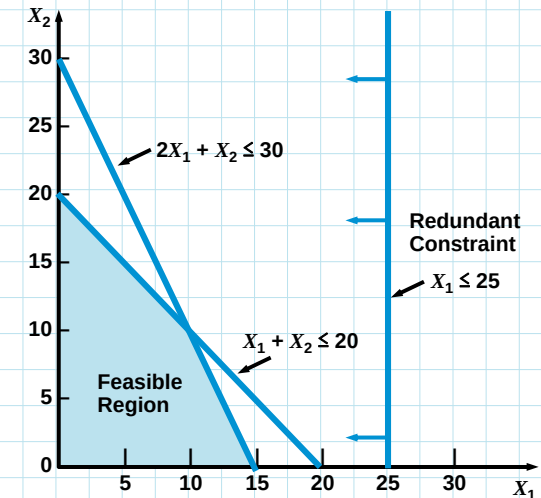


Figure 7.14

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Four Special Cases in LP

- Redundancy

- A redundant constraint is one that does not affect the feasible solution region
- One or more constraints may be more binding
- This is a very common occurrence in the real world
- It causes no particular problems, but eliminating redundant constraints simplifies the model

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Four Special Cases in LP

- Alternate Optimal Solutions

- Occasionally two or more optimal solutions may exist
- Graphically this occurs when the objective function's isoprofit or isocost line runs perfectly parallel to one of the constraints
- This actually allows management great flexibility in deciding which combination to select as the profit is the same at each alternate solution

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Four Special Cases in LP

- Example of alternate optimal solutions

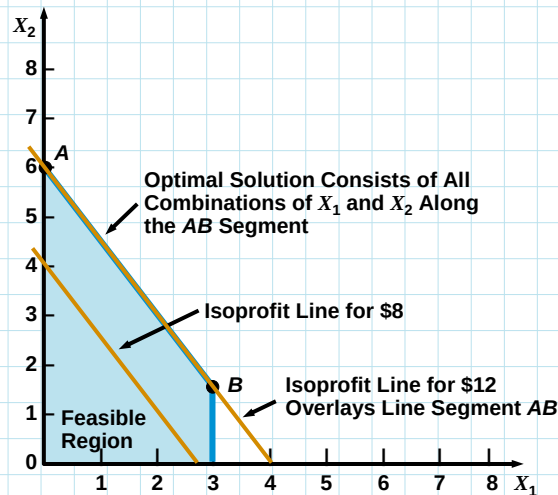


Figure 7.15

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Sensitivity Analysis

- Sensitivity analysis often involves a series of what-if? questions concerning constraints, variable coefficients, and the objective function
- One way to do this is the trial-and-error method where values are changed and the entire model is resolved
- The preferred way is to use an analytic postoptimality analysis
- After a problem has been solved, we determine a range of changes in problem parameters that will not affect the optimal solution or change the variables in the solution

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Sensitivity Analysis

- Optimal solutions to LP problems thus far have been found under what are called **deterministic assumptions**
- This means that we assume complete certainty in the data and relationships of a problem
- But in the real world, conditions are dynamic and changing
- We can analyze how **sensitive** a deterministic solution is to changes in the assumptions of the model
- This is called **sensitivity analysis**, **postoptimality analysis**, **parametric programming**, or **optimality analysis**

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High Note Sound Company

- The High Note Sound Company manufactures quality CD players and stereo receivers
- Products require a certain amount of skilled artisanship which is in limited supply
- The firm has formulated the following product mix LP model

$$\begin{aligned}
 &\text{Maximize profit} = \$50X_1 + \$120X_2 \\
 &\text{Subject to} \quad 2X_1 + 4X_2 \leq 80 \quad (\text{hours of electrician's time available}) \\
 &\quad \quad \quad 3X_1 + 1X_2 \leq 60 \quad (\text{hours of audio technician's time available}) \\
 &\quad \quad \quad X_1, X_2 \geq 0
 \end{aligned}$$

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High Note Sound Company

The High Note Sound Company graphical solution

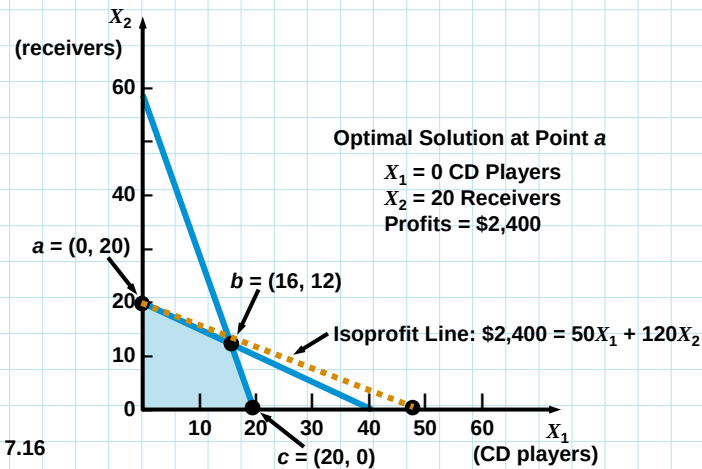


Figure 7.16

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Changes in the Objective Function Coefficient

Changes in the receiver contribution coefficients

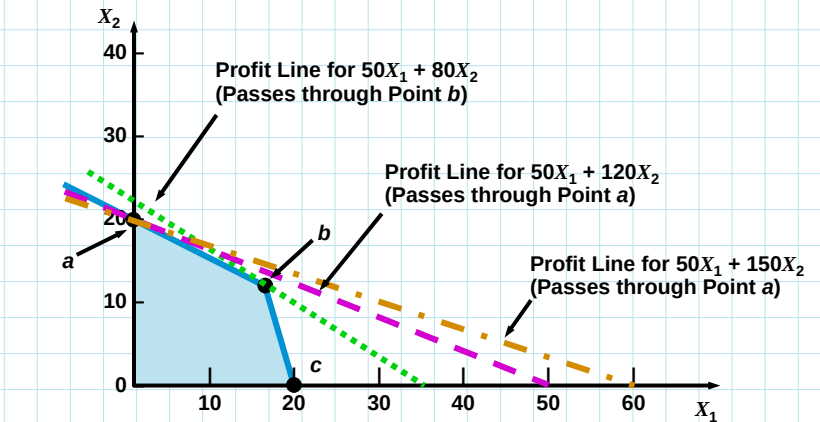


Figure 7.17

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Changes in the Objective Function Coefficient

- In real-life problems, contribution rates in the objective functions fluctuate periodically
- Graphically, this means that although the feasible solution region remains exactly the same, the slope of the isoprofit or isocost line will change
- We can often make modest increases or decreases in the objective function coefficient of any variable without changing the current optimal corner point
- We need to know how much an objective function coefficient can change before the optimal solution would be at a different corner point

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QM for Windows and Changes in Objective Function Coefficients

Input and sensitivity analysis for High Note Sound data

Objective				
☒ Maximize ☐ Minimize				
High Note Sound				
	CD players	Receivers		RHS
Maximize	50	120		
Electrician hrs	2	4	<=	80
Audio tech hrs	3	1	<=	60

Program 7.5A

Ranging					
High Note Sound Solution					
Variable	Value	Reduced Cost	Original Val	Lower Bound	Upper Bound
CD players	0.	10.	50.	-Infinity	60.
Receivers	20.	0.	120.	100.	Infinity
Constraint	Dual Value	Slack/Surplus	Original Val	Lower Bound	Upper Bound
Electrician hrs	30.	0.	80.	0.	240.
Audio tech hrs	0.	40.	60.	20.	Infinity

Program 7.5B

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Excel Solver and Changes in Objective Function Coefficients

Excel spreadsheet analysis of High Note Sound

We will place the answers here.

Calculate the total for the objective function and the constraints based on the product of the data coefficients and the variable values in B4 and C4 using the SUMPRODUCT function.

Enter the names for the objective function and constraints. Note that in this example we have placed the variables above the remaining information. Because Excel is open ended, the sheet can be designed any way we like.

The objective is in cell D6. The cells to change are specified as B4 to C4. Cells D9 through D10 (the left-hand side) must be less than cells F9 through F10 (the right-hand side) respectively.

Program 7.6A

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Changes in the Technological Coefficients

- Changes in the **technological coefficients** often reflect changes in the state of technology
- If the amount of resources needed to produce a product changes, coefficients in the constraint equations will change
- This does not change the objective function, but it can produce a significant change in the shape of the feasible region
- This may cause a change in the optimal solution

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Excel Solver and Changes in Objective Function Coefficients

Excel sensitivity analysis output

The solution values for the variables indicate that we should make 0 CD players and 20 receivers.

If we produce a CD player, our profit will fall by \$10.

If we use 1 more electrician hour, our profit will increase by \$30. This is true for up to 160 more hours. The profit will fall by \$30 for each electrician hour less than 80 hours.

Program 7.6B

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Changes in the Technological Coefficients

- Change in the technological coefficients for the High Note Sound Company

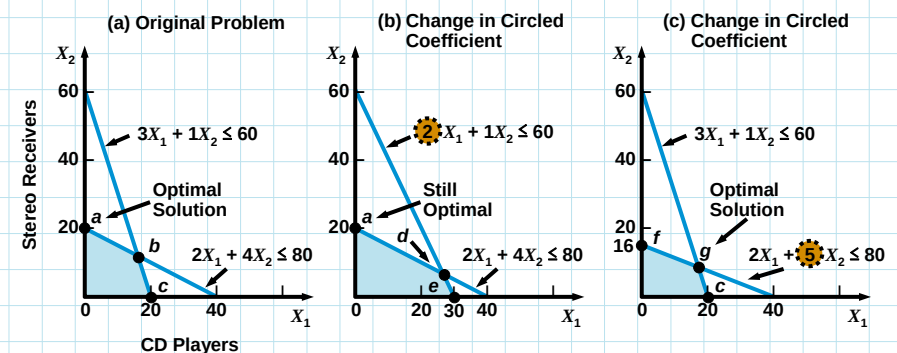


Figure 7.18

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Changes in Resources or Right-Hand-Side Values

- The right-hand-side values of the constraints often represent resources available to the firm
- If additional resources were available, a higher total profit could be realized
- Sensitivity analysis about resources will help answer questions about how much should be paid for additional resources and how much more of a resource would be useful

Changes in Resources or Right-Hand-Side Values

- However, the amount of possible increase in the right-hand side of a resource is limited
- If the number of hours increased beyond the upper bound, then the objective function would no longer increase by the dual price
- There would simply be excess (*slack*) hours of a resource or the objective function may change by an amount different from the dual price
- The dual price is relevant only within limits

Changes in Resources or Right-Hand-Side Values

- If the right-hand side of a constraint is changed, the feasible region will change (unless the constraint is redundant)
- Often the optimal solution will change
- The amount of change in the objective function value that results from a unit change in one of the resources available is called the *dual price* or *dual value*
- The dual price for a constraint is the improvement in the objective function value that results from a one-unit increase in the right-hand side of the constraint

Changes in the Electrician's Time for High Note Sound

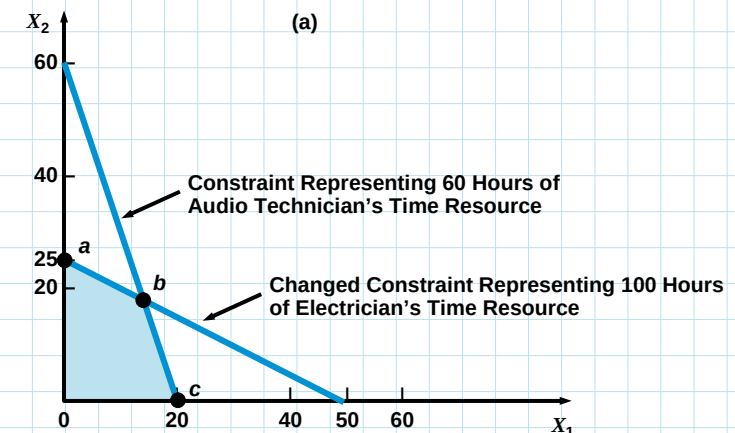


Figure 7.19

Changes in the Electrician's Time for High Note Sound

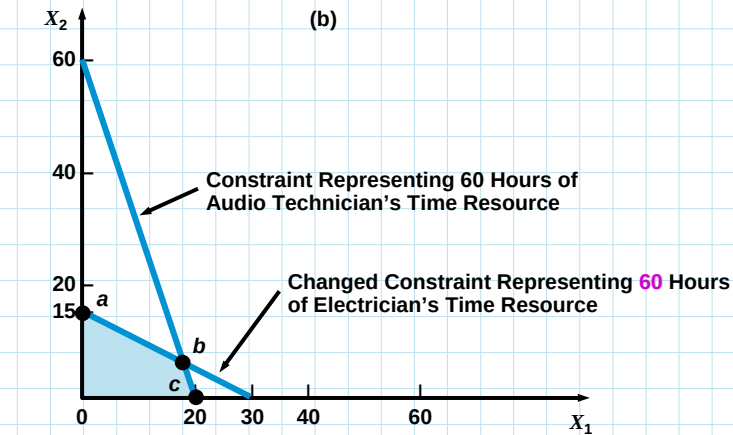


Figure 7.19

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QM for Windows and Changes in Right-Hand-Side Values

Input and sensitivity analysis for High Note Sound data

Objective		High Note Sound		
Maximize				
Minimize				
		CD players	Receivers	RHS
Maximize		50	120	
Electrician hrs		2	4	80
Audio tech hrs		3	1	60

Program 7.5A

Ranging					
High Note Sound Solution					
Variable	Value	Reduced Cost	Original Val	Lower Bound	Upper Bound
CD players	0.	10.	50.	-Infinity	60.
Receivers	20.	0.	120.	100.	Infinity
Constraint	Dual Value	Slack/Surplus	Original Val	Lower Bound	Upper Bound
Electrician hrs	30.	0.	80.	0.	240.
Audio tech hrs	0.	40.	60.	20.	Infinity

Program 7.5B

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Changes in the Electrician's Time for High Note Sound

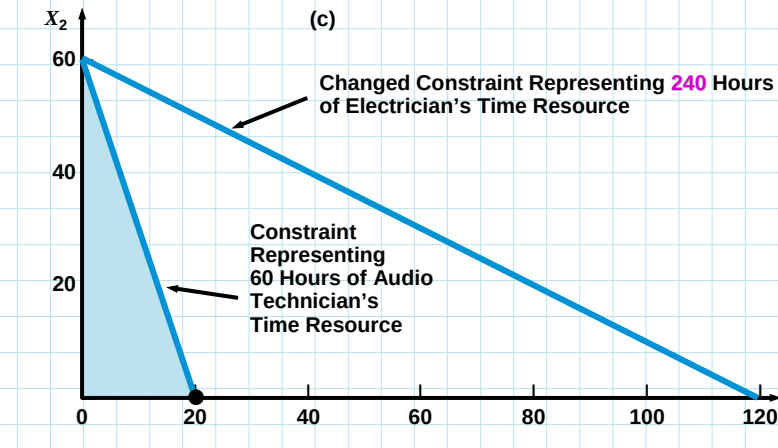


Figure 7.19

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Excel Solver and Changes in Right-Hand-Side Values

Excel sensitivity analysis output

Microsoft Excel 8.0 Sensitivity Report						
Worksheet: [captures.xls]7.6						
Report Created: 8/13/98 10:54:59 AM						
Adjustable Cells						
Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$B\$4	Value CD Players	0	-10	50	10	1E+30
\$C\$4	Value Receivers	20	0	120	1E+30	20
Constraints						
Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$D\$9	Electrician hours Used	80	30	80	160	80
\$D\$10	Audio technician hours Used	20	0	60	1E+30	40

Program 7.6B

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